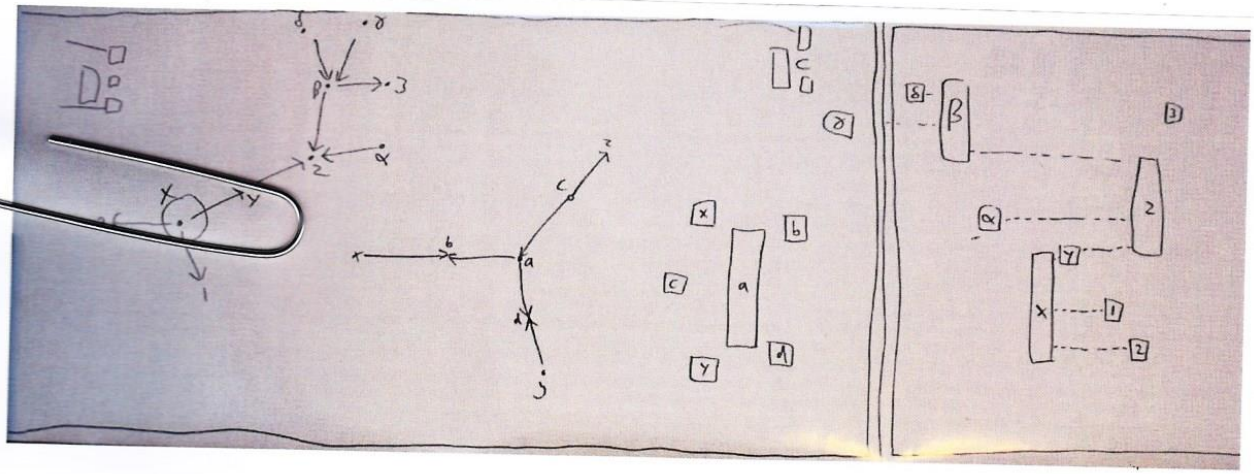

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3.1 Research Meeting: Script for a Conversation (after Imre Lakatos and Douglas Hofstadter)

This conversation takes place in an imaginary classroom, with a variety of relevant printouts scattered across the tables, as well as a laptop. The speakers, X and Y, are non-mathematicians, and are students of linguistics and communication. X has made a recording sitting in on a meeting held between a group of collaborating mathematicians. X and Y are here to discuss a certain excerpt from that meeting, attempting to describe it as a *first time through*[1], which means, rather than thinking of the mathematical objects as the driving force behind what's going on, to shift the focus to what the subjects do to shape themselves into *mathematicians* and *collaborators*, and to produce the work of that evening—the construction of a STATEMENT—as stateable, shareable *work on mathematics*. The speakers also decide to note which resources are needed, how, and when during their process of coming to understand the excerpt.

In their conversation the two are trying to make sense of the excerpt they have from the ground up; the audience has the sense that it is a genuine process of collaborative discovery. The two speakers are at ease, are not afraid to share their uncertainties, and it's clear that although they find the *first time through* approach tricky, they are making an effort to stay on track. They have before them a transcript of the excerpt as well as a reproduction of the board notes as they were at the time of the excerpt. They also have transcripts of other relevant sections of the meeting, printed papers, reproductions of the subjects' movements, and some other printed resources, as well as access to Google, and the points at which these are referenced are noted as clearly as possible in the text. At one point the presenters are joined by a guest, who was one of the participants in the original meeting.

3.1.1 A Layperson's Overview

X: So OK, we're talking about a meeting today. And to set the scene, this was all happening in a … in a room in a university, and it's early evening, and they've got snacks and drinks and, uh, everybody's kind of… talking and laughing a lot. And everybody's more or less oriented toward--there's a whiteboard on one side of the room, and people stand up and write on it and talk about the writing. It seems like… there are two of them who are doing a lot of the talking, and tend to be next to the board so everybody's facing them-

Y: So does it seem like they're… in charge?

X: I mean, not really! People interrupt them and talk over them so it doesn't seem like they're afforded more… power or anything. And it's not all them giving the explanation, you can see F explaining something here (line 08), and I think that's also what's happening with D there (28), and E there (09). And you get C asking a question here (07), and A kind of asking questions to the room here (24) and to B here (31), and they get answered by B and by F, so it's pretty… sort of mixed. I guess not quite equal though. I just get the feeling that… maybe these two people, A and B, they have more to say this time. But everyone's chipping in, so

Y: Right, so there's a kind of… there isn't a clear agreement that a particular person should be listened to most, but instead it's something like, if you feel you can explain something or have a query, you feel empowered by that contribution to speak, something like that? This is tricky. So we're supposed to be describing this as the *first time through*, right?

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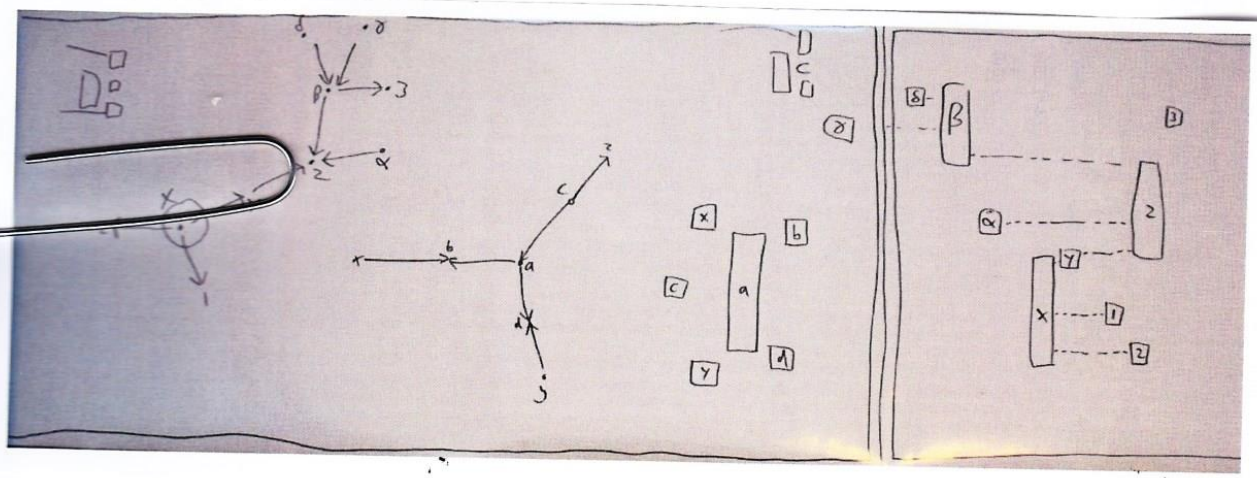
[1] A technique from ethnomethodology, in which an attempt is made to describe what is going on as though it is the first time it has ever happened (Garfinkel *et al.*, 1981)



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30 the whole graph's bad.
31 A: The whole graph's bad?
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X: Yes. Which is hard! We're supposed to try to look at what's actually being done by the people in this room as though it's the first time anyone's ever done this, and try to set aside our preconceptions about what's happening wherever possible.

Y: Yes, that's perhaps even harder… let's see how it goes. It would be nice to try to make a note of everything we find ourselves using to make sense of what's going on, like-- every time we notice ourselves using a new resource, like other papers or things we remember about mathematics.

X: Yes, great. And the group uses different things during the meeting too, like papers and diagrams in their shared Dropbox, and what's on the board, and so on. So we'll start out with this text transcript and our recreation of what was on the whiteboard. So they've drawn some things on the board when this clip starts, there are numbers and letters and arrows, and boxes. They don't actually write anything during this excerpt, so it's just like this. So it starts off with B making a STATEMENT, 'the thing that's bad, is…'

Y: So what's B talking about? 'The thing that's bad…' Is this some kind of Sacksian 'IT'? I'm thinking of the kind described by Garfinkel et al in the paper about pulsar discovery, and attributed to an unpublished lecture by Oliver Sacks before he died – where an 'it' is mentioned, recognised and understood in the course of a conversation while it's still very much an unknown thing, and known to be an unknown thing, and so the word is sort of vague and undefined but is still this very essential tool for the speakers to be able to talk about something and thus come to define it

X: Huh, that's interesting. They do seem to be finding a way to describe the 'thing that's bad', working toward a STATEMENT of a definition maybe. Well, so B says this in the first line, 'the thing that's bad is a vertex with three switching vertices,' and then here (line 07) C repeats it wanting clarification…

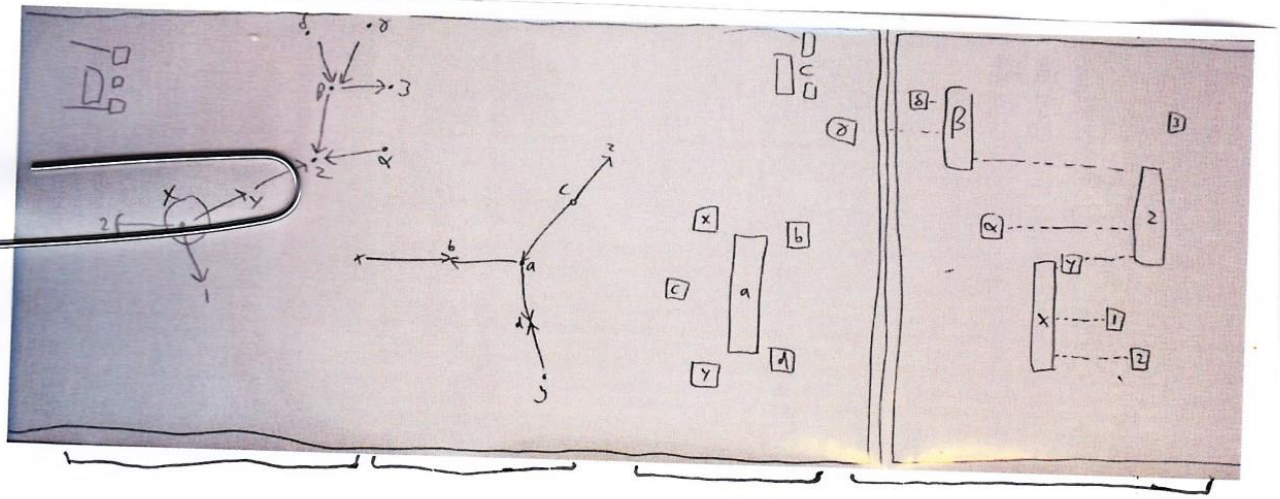
Y: Yeah! C just repeats it in a questioning tone, just reframes the same thing as a question, but we can guess that C's asking B to expand, it's like saying, 'tell me more about this'. So then F starts explaining, and – here (line 12) F is using the word 'we', saying 'we have nowhere…'

X: Right, so we get this kind of collaborative feel to what they're doing here. And in this animation of the gestures we can see B go up to this picture, this diagram here and follow along. And F is mentioning letters, and B is following the lines that are labelled with those letters. So it's connected with the- with this diagram here in some way.

Y: Oh, and so – F is explaining, like describing something, talking through an example maybe to explain what B means- to exhibit 'what's bad', maybe, to show the badness

X: So it's kind of this team effort, between F and B there's this shared understanding, and they can – they want to explain it in multiple ways at once, with words and waving at the picture, and it's important to them to share their understanding with C. And this understanding, it has to do with this – they want to convey it by talking about this picture, it happens between them and the picture. And each individual needs to have a part in this shared understanding. OK, so we have this sense that in this first section B has proposed some kind of STATEMENT of a definition for 'what's bad', and C asked for clarification, and F and B worked together to sort of… *exhibit* that diagram in the centre, as providing support for that definition. I don't think we're quite able to see how that works, yet.

Y: Agreed. Well, let's look at what happens next.



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X: Then things get more complicated, because then we have – so on line 14 here A says, 'I have a problem.' And everything gets kind of chaotic, there's laughing, it's obviously quite informal. And then A walks up to the board and starts pointing at this other diagram [points to diagram on the left]-

Y: Wait, why are you saying 'diagram'? And how can you tell that they're separated out like that?

X: Yeah good question. Uhh… so they just… *look* like diagrams. What does that mean… They're simple, and there are arrows and relationships between elements. I've seen things like this before, called diagrams. And there are labels, you get points, dots, and those have a recognisable character next to them so you can easily refer to them, like they do in these two lines, using the labels. So there are – it's like a network. And there are kind of… four collections of connected things here.

Y: So what do you mean by 'diagram'? What is it you think these images are *doing* here?

X: Umm, I guess they seem kind of like tools for reasoning, like an image that is useful for organising thought, rather than… you know, having an aesthetic purpose. It's something that people can *use*, so you interact with it *as* you're – you're thinking, or you're talking

Y: Or if it were an engine diagram, as you were working on it

X: Right! So they have this functional aspect, and the way they are drawn is supposed to make something easier, somehow.

Y: OK, good. So… so A walks up to the board…

X: Right, and starts pointing at the *other* diagram, waving arms around it, it seems as though A is talking about it, directing action at this diagram. And it sounds like a continuation, not a change of topic.

Y: OK, so B makes this one clear STATEMENT at the beginning. And then A says 'I have a problem'. And this is announced with the same kind of gravity, emphasis, loud voice, waiting for people to stop talking, that kind of thing, so it seems kind of like a statement put on the same level as B's first one, like it's a response maybe. But then A is not pointing at the same diagram.

X: Hm. So A *could* be exhibiting 'badness' in the other diagram, but that doesn't seem right, somehow. It's more like a contradiction.

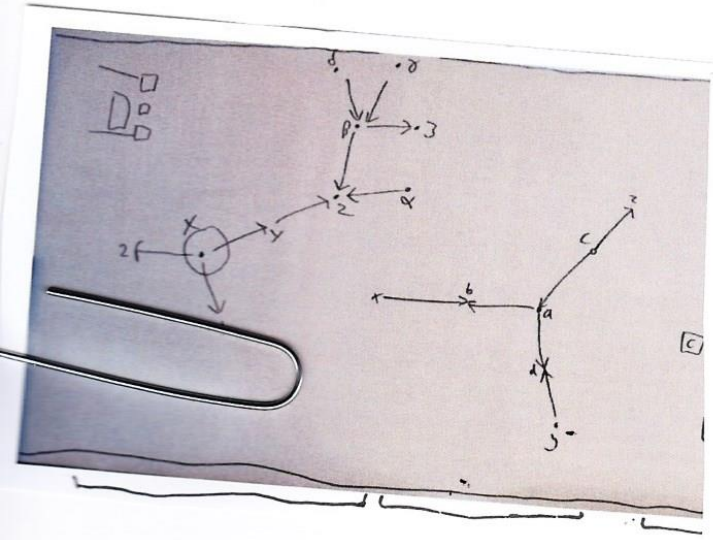
Y: Right. So … so they're talking about something that somehow exists between the diagrams, perhaps the *bad thing* is *in* one and not in the other, and that's what they want to talk about

X. Yeah.

Y: OK, and this announcement has quite an impact, right? B says 'oh god', E says 'not again'…

X: Yeah they're acting like it's a disaster! And then they say, what was it… 'here comes another one of A's counter-examples.' But they're not serious about that, they're kind of doing this cartoon 'oh god' response, like they pretend they hate A for it but really this kind of setback is just part of the usual course of things

Y: Setback? Why that?



ique document, on file at the Center for History and Philosophy of Physics at the American Institute of Physics, was made available for our examination.[4] The tape was transcribed by us using the conventions of conversational analysis.

Our question was: 'What does the optically discovered pulsar consist of as Cocke and Disney's night's work?' The tape and transcript permitted us to treat some relevancies that are not otherwise available in science studies:

(1) That the discovery as their night's work had the property of 'first time through'.

(2) The local historicity of the night's collection of observations.

(3) The quiddity of their night's work.[5]

We tried to respect these properties of their night's work in asking what their discovery could be and so what we entertained their discovery to consist of may seem strange. With the hope of making it plain we begin by characterizing their discovery with the metaphoric use of a 'gestalt theme'. Their discovery and their science consists of astronomically 'extracting an animal from the foliage'. The 'foliage' is the local historicity of their embodied shop practices. The 'animal' is that local historicity done, recognized, and understood as a competent methodic procedure. The 'animal' formulates their embodiedly witnessable astronomical competent practices as the transcendental properties of the independent Galilean pulsar.[6] Their science consists of the optically discovered pulsar as the produced practical observability of their ordinary night's work.

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X: Umm, I guess I get the sense that this is B suggesting something, and then A coming back with something that contradicts it. So it- derails what the first person said, it messes up that suggestion, but it advances the group's work in the process because it shows that there was something they weren't seeing.

Y: So we think that – it isn't just a counter-argument, it might really disrupt things

X: Right, it isn't like a difference of opinion.

Y: OK, so then what. Then things go kind of quiet, and A has trailed off, just kind of pointed and trailed off, and there's a silence. And then on line 26 here B says, 'the problem is not for x', and A says OH! Like, it seems like there's some revelation, like B has talked A into something.

X: Yes! Yes, that seems right, like that… saying 'not for x' is enough to completely change A's understanding somehow. But I don't yet see how! OK, well then what. Then B repeats the statement-

Y: Why does it seem like a statement?

X: Mm, I dunno, the phrasing has a kind of formality to it, starting with 'if there exists' like that. It sounds like something you'd read written down in a paper, whereas everything else is so much more informal. It's kind of a switch in tone.

Y: Oh, and you think the same is true of the first line?

X: Yeah, that seems less formal, I guess. There's a little of that in line 3, that it's so… specific, so technical. But also in line 1 when it's introduced B starts off with 'alright so', it's kind of like – fanfare

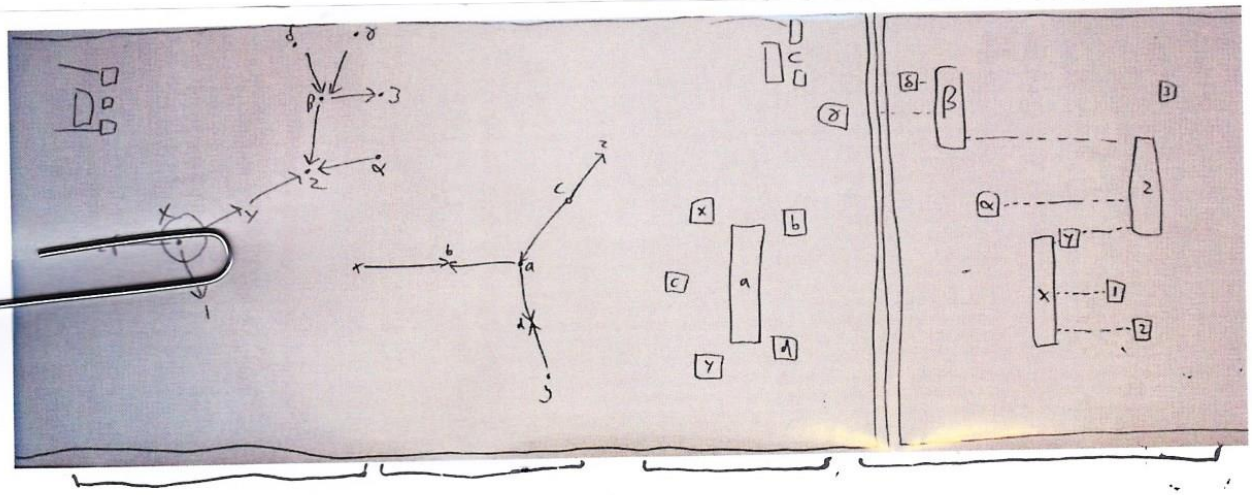
Y: Sure. And this is perhaps… in that Garfinkel *et al.* paper (1981) they talk about identifying what the group does to extract the animal from the foliage, which roughly speaking is extracting 'the *scientific work*' from their various doings that night. So in that paper, they argue that atemporal properties of the pulsar were still established as an exhibitable thing by people doing things with equipment one night. So here, we want to see what they do to extract the *mathematics* from their exchanges, sketches and so on, the *thing with wider use to the community*, that's produced according to the requirements and aims of the mathematical world. This fanfare seems like it's signalling something.

X: OK, so- maybe we should see what they work toward here, and what principles are employed. OK, so some general thoughts. They're talking about how to define what's 'bad', I think, especially B's lines at the beginning and the end. And then right at the end, A repeats what B has said, asking for an explanation, and B expands that 'bad', B replaces it with 'the tree cannot be represented'. So I think maybe when they talk about what's 'bad', they're talking about cannot be represented, that there's a representation that you can't build for those cases.

Y: Yeah, that makes sense. So one part of what they're doing is… the group wants to be able to represent the tree, and not being able to do so is 'bad'.

X: Right. And in another sense their work is to find a STATEMENT of a definition for what's 'bad'.

Y: OK. So what are they… talking about here?



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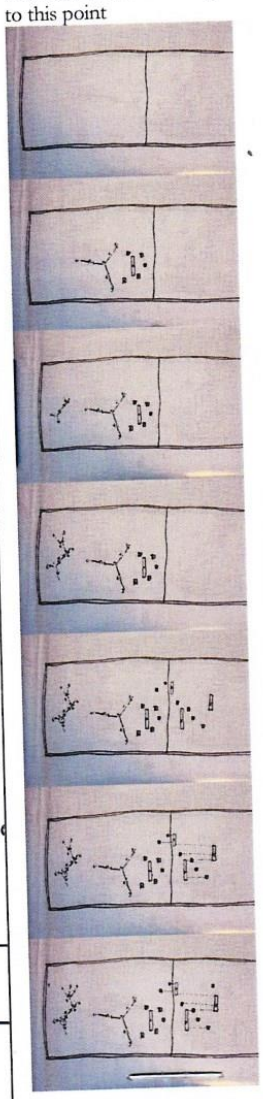
Chronological record of what was on the board up to this point



X: Mm, I have some guesses, but we should mark this as bringing in a bit of general knowledge about the mathematical world, because some of this has to do with the terms they're using.

3.1.2 Basic Mathematical Knowledge

Y: Alright, well we both know a bit from experience about mathematics, from school and so on. Can we bring any of that to bear?

X: Yep OK. They're talking a lot about vertices and graphs and paths. A vertex is kind of like a corner, right?

Y: Right, an endpoint [draws dot], or like where two lines meet [draws corner]. And the mention of graphs, I don't know much about graphs but I guess I could imagine these two diagrams being graphs. And I know there's a field of study called 'graph theory'. So if there are people out there studying graphs and how they work, maybe that's what these people are doing.

X: Yes! Oh and look at the other types of drawing over on the other side, there are two basic types of drawing here, and maybe they're studying how they relate to each other.

Y: Right, but I think we'll have to look a bit at the rest of the meeting to understand that. What else do we have for mathematical terminology?

X: Oh, 'counter-example'! On line 18. A counter-example is normally something that you… that stands against something you're arguing for, that contradicts what you're saying by the fact that it exists. So when B says that 'here comes a… counter-example'… maybe that means B's imagining that A is about to come out with an example that contradicts B's original STATEMENT. And it … it kind of sounds like A has played that role before, they say 'another one of A's counter-examples'.

Y: Ooh, that's helpful. And thinking about examples, and counter-examples, might help us to make sense of what the diagrams are there to do, if they're both being discussed in the course of one piece of work.

3.1.3 The Rest of the Meeting

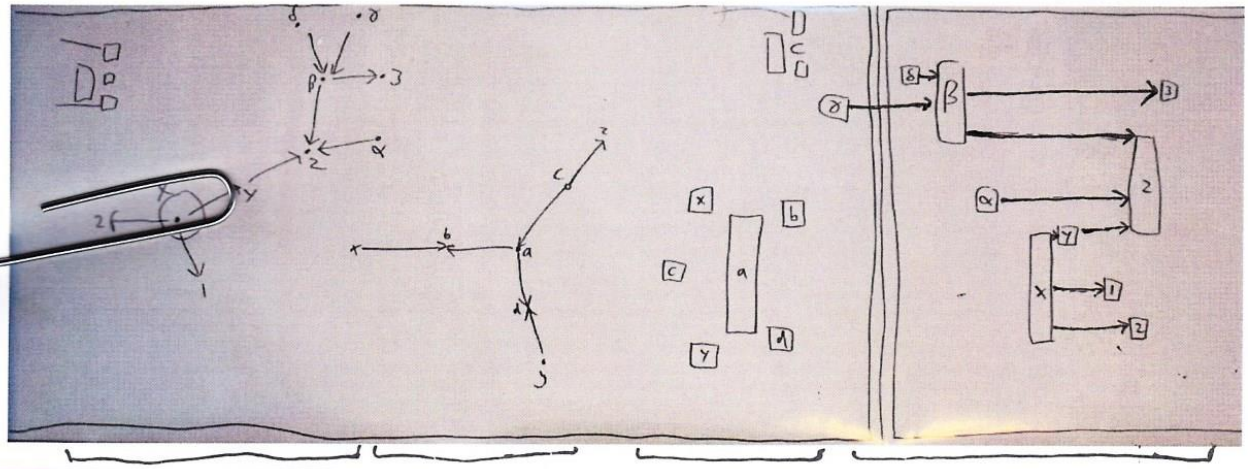
X: Alright, so let's think about how much more we can work out on the basis of the rest of the meeting. So really just the conversations that were had on that same day, in the same room. The first thing is these diagrams, some of them showed up at the same time as each other. You can see that in this chronological record of what was written on the board when up to this point.

Y: OK, yeah. So we can see that… There are two types of drawing, right, one with arrows and one with rectangles. And these two show up at the same time next to each other, and the two on the outside show up at the same time, on either side, each one on the side – arrows on the left, rectangles on the right. As well as the timing, we can see that the letter labels we get here, the a, b, c, x, y, they all appear in both of the inner diagrams. Oh, but no z?

X: Aah, you remember, F said 'we have nowhere to put z', on line 12.

Y: Ohhh, right. Ooh. What does that mean, I wonder. And the same for the other two, we have x, y, z, 1, 2, 3, α, β, γ, δ… wait, what? what a mixture…

X: Yes, I think I remember them laughing about this! It's an unspoken rule, normally you'd choose one alphabet or numbering system and stick to it, like Greek letters or Arabic numerals



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B: So we were hoping – we were hoping – that – in this step of the algorithm, °nv the proof°, regardless of what oriented path cover ((gestures to 'tree' on left)) we could- we could js convert it into a rectangle visibility graph ((gestures to rectangles on right))…. um… which is false.
A: At least the idea of making the paths be rows in the …
B: [((unintelligible))
A: [the whole point was the coherent paths were supposed to be reading left to right
B: if you start with the fundamental idea of … the arrows are always pointing from left to right in the representation
A: yeah

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→ 09 E: [((unintelligible)) right there
10 F: … *b* is switching for *a*, [*d* is switching for *a* [and *c* is supposed to be switching for *a*
11 B: ((walks over, follows path *xba*)) [yup ((follows path *zca*))[yup
12 F: but we have nowhere to put – *z* ←
13 B: right
14 A: I hav- I have a problem.
15 B: oerr gahd
16 B: OK what's your problem ((laughing))
17 E: ((laughing)) not again
18 B: ((speaking through laughter)) here comes another one of A's counter-examples
19 A: it just seems… ((points at diagram on board)) isn't… wait, what happened to this… so
20 A: so… *z* was switching. … [but what if it just continued this way and it can switch ((follows path))
21 B: [Yuh. I-
22 A: but then a couple of things switched all at the- ((mimes two coming down from above))
23 B: so if there was
? → *24* A: °s that a problem?°
25 B: if there was… ((walks over to board, A steps away))
26 B: the problem is not for *x*
27 A: O↑H↓
→ 28 D: ((unintelligible)) rectangle
29 B: if there exists a: swi- a ↑vertex↓ from which three vertices are switching… then it's ↑bad↓. then
30 the whole graph's bad.
? → 31 A: The whole graph's bad?
→ 32 B: the- the tree cannot be represented if the vertex ((trails off))

or whatever. You use these systems, so that everything gets a different label but they're all taken from the same set so that they're all kind of… equal, you show that all of the vertices or whatever are of a kind. But they kept mixing them up. And it really cracked everybody up, because it was so against convention.

Y: That's funny. They're supposed to be all of a kind, and just messing with that a little makes them so much more visible as markers.

X: OK so I'm getting the impression that for each arrow-graph there's a rectangle-graph, they come in pairs like that.

Y: Right. So what's the relationship?

X: Uhh, I think this comes up, early on in the meeting. Yeah, here, about 7 minutes in. They talk about… they were *hoping* they could take an 'oriented path cover', and that was something B said while pointing at one of the trees… and thinking they could 'just convert it into a rectangle visibility graph'. So there's some kind of conversion going on here.

Y: Right, great. So you're thinking that they're saying that… if you had a tree diagram, an 'oriented path cover', then it might be possible to draw a rectangle visibility graph that was somehow equivalent to it. An equivalence that has something to do with those labels that we were noticing were the same. And looking at where those labels are, it looks like maybe each character [points to x, y, z in outer two diagrams] is assigned to a point, a vertex, in the oriented path cover, and to a rectangle in the rectangle visibility graph. And in the tree diagram, they're connected by arrows, and in the rectangle one, they're connected by dotted lines…?

X: Yes and I think there's something about that here too. Further on in the same excerpt, here, they keep talking about *going from left to right*, and they say 'arrows are always pointing from left to right in the representation'. So while one type of diagram [points to tree diagram] has… directionality, the connections between the elements are directed, the other type [points to rectangle diagram] maybe it's kind of built in to their relative positions. Let me show you by drawing over the dotted lines [draws arrows from left to right on rightmost diagram], I think it's about which is lined up with which, so that… it says 'visibility', it's about which rectangle can 'see' which other rectangle if they're all facing to the right…

Y: Except that rectangles don't have faces

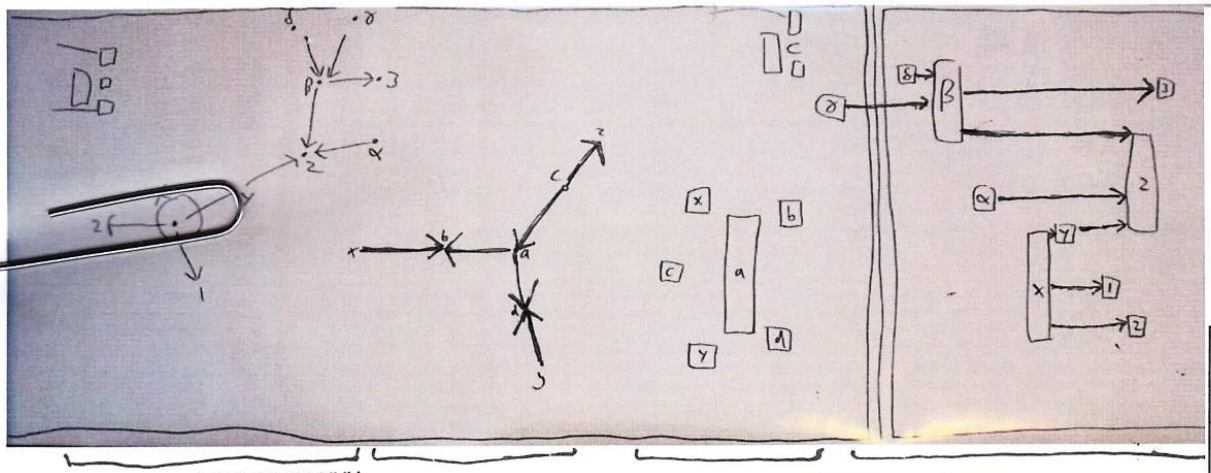
X: Except that rectangles don't have eyes, right. But it still makes sense, right? If you figure out who can see whom, and who's to the left, then it matches up here with the arrows.

Y: It's funny, I get what you mean about them 'seeing' each other but in the actual diagram they've drawn, they barely line up, or *do* line up when they aren't meant to

X: Yes, that doesn't seem to matter. The representation, it doesn't have to actually have the property.

Y: Why not? Surely we want it to?

X: Well, I guess the point is that it's embedded in a discussion, everybody's talking about it and… the diagram doesn't stand alone, it's something that happens in combination with a bunch of acting people



00:55:44.000

01 B: alright so the thing that's [bad
02 D: [OK I'm with you
03 B: … is a vertex with three switching vertices
04 C: [what
05 A: ((nods)) [or more
06 B: or [more
07 C: [a - vertex with three switching vertices?
08 F: cos like [in that with that vertex *a*
09 E: [((unintelligible)) right there
10 F: … *b* is switching for *a*, [*d* is switching for *a* [and *c* is supposed to be
11 B: ((walks over, follows path *xba*)) [yup ((follows path *zca*))[yup
12 F: but we have nowhere to put – *z*
13 B: right
14 A: I hav- I have a problem.
15 B: oerr gahd
16 B: OK what's your problem ((laughing))
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30 the whole graph's bad.
31 A: The whole graph's bad?
32 B: the- the tree cannot be represented if the vertex ((trails off))

B: so. ((long pause)) how bout this <For this vertex x … u::hh … the vertices are switching or whatever you wanna call it … if … you have a path uuuf directed in one way from x? so that vertex… > and then at least one edge directed ((unintelligible))
((explains wrt concrete example))
B: so x has one switching vertex fr… it
((Switch to talking about vertices being switching))
A: are you doing what I think you're doing? so you're talking about VERTICES being switching. VERTICES are being switching. and then you <limit how many switching vertices there are>.

?: I dunno. Just if there [are no]

A: [just if there] are no k-bad vertices for >k greater than or equal to< THREE … then… t exists… >oh wait< then…

B: then this algorithm works to convert this orienting path cover into a rectangle

E: algorithm to be written

Y: Right; it's kind of like an ASSEMBLAGE, made up of people and environment. Right, so then when B says something about being able to 'convert it into a rectangle visibility graph' on line 38, that might be what that's all about.

X: Right. So then… I'm going to try it with the other graph. So we have a starting rectangle, [points to biggest rectangle on middle diagram, then points to each smaller rectangle] and then three more, and then I can put x here, and y here, and…

Y: And there's nowhere to put z! Just like F said on line 12!

X: Right, and now we're getting a bit of sense to the… the idea that it isn't possible to represent the graph sometimes, and this middle diagram here, it's there as a counter-example

Y: One of A's counter-examples!

X: So this is the original counter-example. A counter-example that means that you can't *always* represent a tree graph as a rectangle graph, as they had supposed. And I guess that when B is making these STATEMENTS about what's 'bad', they are trying to define which tree graphs this won't work for. So in this diagram [points to middle diagram], there are three arrows coming out of one point, and then each goes on to another arrow, and the arrows are pointing in different – one goes out and the other goes in, or one goes in and the other goes out. So they have to be one block to the left of the other and the next to the right, and vice versa. And you can do that on either end of the first rectangle, but after that, the third time, there isn't anywhere to put it…

Y: So you think that… that the basic aim here is to try to – you're calling this a counter-example, and you think that it's an effort to somehow write down… what it is …

X: A description of some property of the counter-example that is connected to why it doesn't work

Y: *Why* it doesn't work? What do you mean by that?

X: Oh wow, I guess I don't know!… I think I mean… OK, how about a description that would help you to know whether another example would work or wouldn't, or would let you know how to construct one that wouldn't work. How's that?

Y: Hm. It's… OK. I guess if we feel like we understand something, we feel like we can predict it.

X: Yeah. So I think… judging by B's attempts to make these neat STATEMENTS, what they want is to write a kind of description… here it's a verbal description, a description of the… the *non-working thing* in the counter-example.

Y: Yes, and it's stated in this emphatic way… it's very brief, just a short sentence

X: Right, so the exact wording is important. Actually, you can see similar short phrases repeated during the meeting, here, and here, and here… it's almost like they're adjusting it. So maybe… adjusting the wording of the phrase they keep repeating is… somehow keeping track of the shared understanding, of the way the group is seeing the example, in this portable phrase that can be easily repeated

Y: Tell me more about these adjustments.

00.30.48.000

B: so. ((long pause)) how bout this. <For this vertex x … u::hh … the vertices are switching or whatever you wanna call it … i:f … you have a path uuuf directed in one way from x? so that vertex… > and then at least one edge directed ((unintelligible))

((explains wrt concrete example))

B: so x has one switching vertex from it

((Switch to talking about vertices being switching))

A: are you doing what I think you're doing? so you're talking about VERTICES being switching. VERTICES are being switching. and then you <limit how many switching vertices there are>.

00.16.20.000

B: ((finishes drawing diagram)) So- does that count as one- so I'm looking at that vertex ((circles x)). Does that count as one switching path or two

00.18.04.000

F: Yeah so what do we mean by a switching path
E: Yeah so what's a switching path
A: WELL UH- TH- K- OK. I was hung up on this also. It doesn't mean <path of the path cover>. It's [just
C: [Oh
A: a path in the graph RELATIVE TO the path cover is switching.
A: So this is a path in the graph ((follows x to δ))

00.18.46.973

F: So whether we have to channel from whatever vertex we're looking at all the way to a leaf. … before we decide whether it's switching.

00.20.05.000

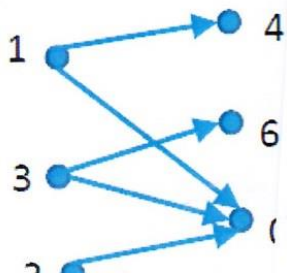
D: So am I right for that vertex we're saying that's- we got THREE switching paths?
C: Three- well we don't know that
A: ((following paths to ends. follows x to δ)) one… ((follows x to α)) two… ((follows x to γ)) three. Yes
B: I- uh- does it
C: Cause you go all the way to the leaf.
F: I mean if- I mean IF, we haven't- determined that
A: Or it has just one!

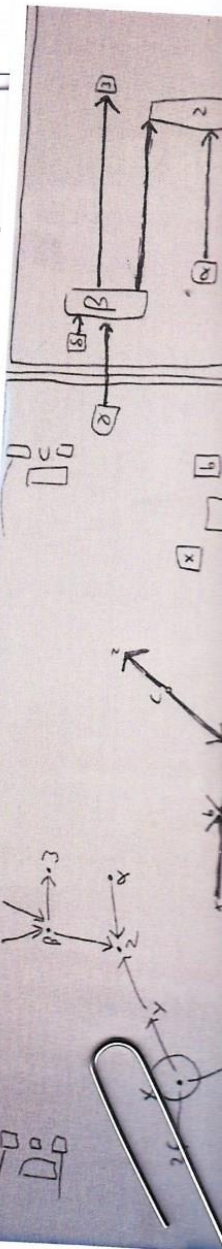
Seems hard to add these paths as "rows" in an RVG

1				4		
1			0			
	2		0			
		3	0			
		3				6

(I'm having a hard time using the OPC approach that we discussed to include the (1,4), (2,5), and (3,6) arcs around the rectangle for vertex 0.)

Can we find a fix for this kind of thing?





X: Alright, so right now B is saying that… 'the thing that's bad… is a *vertex* with three switching *vertices*'. Earlier on we see this line here that talks about switching vertices. There's a *change* here. Before this line the group had been talking about switching *paths*, so – I think by that they mean… if you follow a line of arrows, you follow one to a point, then the next, then the next, like this. So then the word 'switching' refers to the direction of the arrows, whether it changes direction as you go along.

Y: Wait, what do you mean by 'changes direction'? Relative to what? I understand what you mean by 'as you go along', of course, but why should it matter – I mean, it's changing direction relative to what?

X: Yeah, I'm not sure. I guess I'm just talking about—I'm talking about when two arrows point away from one another, or toward one another. Actually I think this is discussed earlier in the meeting, at around 16-18 minutes in [finds excerpt], they talk about what counts as a switching path. It's relative to a thing they call a path cover, which I *think* is what decides where those arrows are pointing, like on this [shows diagram from Dropbox]- this is something from their Dropbox, which I think is the first presentation of this middle diagram here, of the first thing they had trouble representing.

Y: Right, I think I see that, but what about where you… *count from*, if you see what I mean?

X: Well, I think you can start anywhere, actually. F says here, 'from whatever vertex we're looking at'

Y: OK. And then what counts as a path, a string like this [follows path along on leftmost diagram]? And what happens if it branches?

X: Right, you have to decide where it ends and how to count them.

Y: Oh I see, so whether – whether a path has to end at a 'leaf', which I take it is one of these endpoints here. And how many times you count it if it later branches.

X: And then there's a shift, here, to talking instead about switching *vertices,* the first time that a path switches. Look, they emphasise it, it's a total shift in focus.

Y: Oh, wow! OK, so by changing that word a person is changing the focus from… rather than talking about a path, which might go from point a to point b or to point c, they're talking instead about the switching being sort of *decided at one point*, where the arrows point together or apart. [points to first switch in leftmost diagram] It's localised at the first moment that it happens, as you encounter it coming from a starting vertex.

X: Right, so the switch in focus, it subtly evades those questions. And then I think that the 'vertex with three switching vertices' that B talks about, or it's a little clearer at the end here when B says '↑*vertex*↓ from which three *vertices* are switching', that means… B's emphasising that first 'vertex', trying to say that it's a question of whether you can find *any vertex anywhere* with three of those first-time switching vertices, and we just have to look for the first time it happens down a path, so the rest of the path isn't important. It changes what you look for. And it's not just the content of the statement, it's where the emphasis is placed.

Y: and so then we can follow this, and say… *c* is switching for *a*, *d* is switching for *a* – just like F said!

Determine the activities of the family prior to visitor's arrival

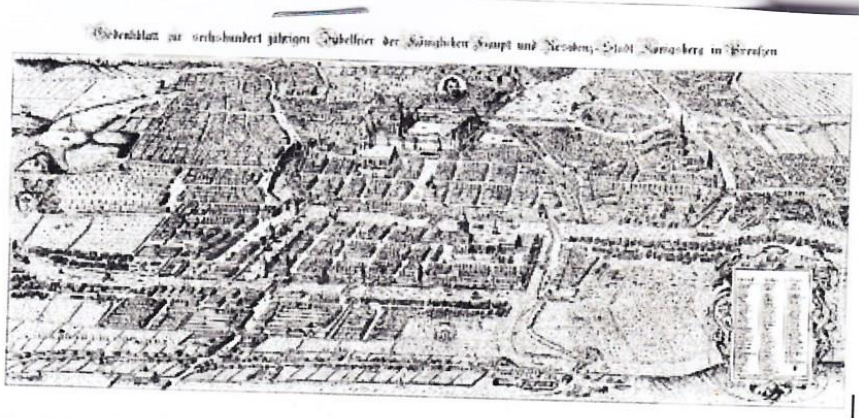
6.

Remember t characters' clothes

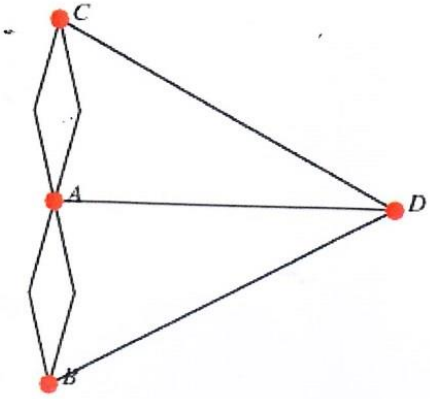
7.

Surmise how lo the 'unexpected visitor' had bee away.

All maps are from Kaikus (Kaikus, 1973), superimposition from (Archibald, 2008).



Koenigsberg. Map by Bering 1613



Representation as a graph

On Rectangle Visibility Graphs

Prosenjit Bose[1], Alice Dean[2], Joan Hutchinson[3], and Thomas S

[1] Université du Québec à Trois-Rivières
[2] Skidmore College
[3] Macalester College
[4] Simon Fraser University

Abstract. We study the problem of drawing a
that the vertices of the graph are rectangles th
axes, and the edges of the graph are horizontal
Such a drawing is useful, for example, when
contain information that we wish displayed
to write this information inside the rectang
We call a graph that can be drawn in
graph, or *RVG*. Our goal is to find clas
We obtain several results:
1. For $1 \leq k \leq 4$, k-trees are RVG
2. Any graph that can be decom
3. Any graph whose vertices
RVG.
4. Any graph with maxim
independent set is an RV
Our proofs are constructi

1 Introduction

In this paper we consi
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Google Scholar

rectangle visibility graph

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On rectangle visibility graphs
P Bose, A Dean, J Hutchinson, T Shermer -
We study the problem of drawing a **graph** in th
rectangles that are aligned with the axes, and t
vertical lines-of-sight. Such a drawing is useful, fo
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Rectangle-visibility representations of bip
AM Dean, JP Hutchinson - International Symposium on G
The paper considers representations of bipartite **graphs** as
graphs whose vertices are **rectangles** in the plane, with adja
and vertical **visibility**. It is shown that, for p≤ q, K p, q has a re
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On **Rectangle Visibility Graphs**. III. External **Visibi**
TC Shermer - CCCG, 1996 - books.google.com
Let'R = be a collection of pairwise disjoint closed **rectangles** in the plane
and R,"will be called visible if there is a closed nondegenerate rectangular
a band of **visibility**) such that one side of ng is contained in a side of R5, th
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Rectangle-visibility representations of bipartite **graphs**
AM Dean, JP Hutchinson - Discrete Applied Mathematics, 1997 - Elsevier

X: And if there are three switching vertices coming off of one vertex, then we have nowhere to put it!

Y: This use of 'vertex' to focus attention on the beginning, it reminds me of those eye tracking studies of Alfred Yarbus', showing how attention is directed when people are asked to answer certain questions about a picture. Those sort of… it's like a map of perception being active and guided by the mind, guided by what you're looking for even if you aren't that aware of it. So in a way these subtle shifts in wording and emphasis are sending the group's eyes and minds around the diagram, paying attention to different parts. They're an ATTENTION-DIRECTING TOOL, and the diagram can be a very different thing depending on how you look at it.

3.1.4 The Mathematical Community

X: OK, so we think this connects up with graph theory, maybe

Y: Yeah. So that's… have you heard of the Königsberg Bridge Problem?

X: That's the Russian city with an island, and a river, and seven bridges, right?

Y: Right, and the problem is trying to find out whether there's a route around the city that would cross each bridge only once. And nobody could find one, but nobody had been able to demonstrate that it wasn't possible at all until Euler found a way to study this and prove it in the 1730s. I think it was…

X: OK, so this is supposed to be a kind of… graph theory representation, right? With, uh, vertices and connecting lines. So all the landmasses are vertices, and each line is a bridge-route.

Y: Yeah, so then a city is represented as a graph, and then it's possible to… to say what can and can't be done with that graph. It's the study of these constructions.

X: Yep. And I think… they mention rectangle visibility graphs, somewhere…

Y: Oh, great! So if I search that phrase on Google scholar, then I get some results. Bose, Dean, Hutchinson. There's this one, Bose, Dean, Hutchinson, Shermer (Bose et al., 1997). They say 'Our goal is to find classes of graphs that are RVGs.'

X: OK, so it seems like people are concerned with working out what can and can't be represented. But it looks like these are just with lines, not arrows.

Y: OK, so there are a few things that look different and we don't know why? Shall we ask C?

X: Yes!

C: So… there was something called bar visibility graphs, and then interval graphs. Rectangle visibility graphs were an extension of bar visibility graphs. I can give you some papers … there's a paper here on rectangle visibility graphs from about '97, this is Dean & Hutchinson, 1997, which is probably roughly as old as the field is…

X: Oh, great! So we have here… a couple from the '80s on bar visibility graphs, which are Wismath, 1985; Kirkpatrick & Wismath, 1989

C: Yeah, so they're a little different - for the bars it's literally just bars like this [draws lines], and then you only get visibility in one direction. Rectangle visibility you have both, [draws rectangle]

On Rectangle Visibility Graphs

Prosenjit Bose[1], Alice Dean[2], Joan Hutchinson[3], and Thomas Shermer[4] *

[1] Université du Québec à Trois-Rivières
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[4] Simon Fraser University

Characterizing Bar Line-of-Sight Graphs

Stephen K. Wismath

University of Lethbridge

Weighted Visibility Graphs of Bars and Related Flow Problems (Extended Abstract)

David G. Kirkpatrick *Computer S...

ELSEVIER

Discrete Applied Mathematics 75 (1997) 9–25

DISCRETE APPLIED MATHEMATICS

Rectangle-visibility representations of bipartite graphs

Alice M. Dean[a,1], Joan P. Hutchinson[b,*]

interior points. In this situation, we will call two rectangles u and v *visible* if there is a *band of visibility* $B_{u,v}$ between them: $B_{u,v}$ is a rectangular region with two opposite sides that are subsets of u and v, and such that $B_{u,v}$ intersects no other rectangle of $\mathcal{R}$. The *visibility graph* of $\mathcal{R}$ is the graph of the visibility relation on the elements of $\mathcal{R}$. We call a graph G a *rectangle-visibility graph* or RVG if it is the visibility graph of some collection $\mathcal{R}$ of rectangles; in this situation, $\mathcal{R}$ is called a *layout* of G. The edges of a rectangle-visibility graph G can be partitioned into the two sets representing horizontal and vertical visibility; each of these two edge sets forms a BVG. Thus G, as a union of two planar graphs, is said to have *thickness-two*. Much less is known about thickness-two graphs than about planar ones, although their recognition is known to be NP-complete.

Wismath [14] has shown that every planar graph has a rectangle-visibility layout. Hutchinson, Shermer, and Vince [8] show that a rectangle-visibility graph with n vertices has at most $6n - 20$ edges, in contrast with thickness-two graphs, which may have at most $6n - 12$ edges; in both cases these bounds are attainable. Dean and Hutchinson [4] show that $K_{5,5}$ is *not* a rectangle-visibility graph though $K_{5,5}$ plus any edge is; see Figure 2. Thus rectangle-visibility graphs are not closed under the formation of subgraphs.

Each of the classes of BVGs and RVGs has two important subclasses: graphs with *noncollinear* layouts and those with *strong* layouts, as defined below.

2.3 Collinear and noncollinear layouts

A bar-visibility layout is called *noncollinear* if no two line segments have collinear endpoints; a rectangle-visibility layout is noncollinear if no two rectangles have collinear sides. The 4-cycle (see Figure 1a) does not have a noncollinear bar-visibility layout; Figure 3 shows a (collinear) rectangle-visbility layout of $K_{4,4}$ minus an edge; by a result in [3] it has no noncollinear layout, but a noncollinear layout of $K_{4,4}$ minus two edges is shown in Figure 10.

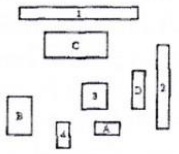


Fig. 3. A collinear (and strong) layout of $K_{4,4} - e$.

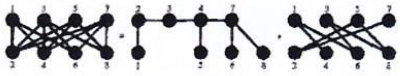


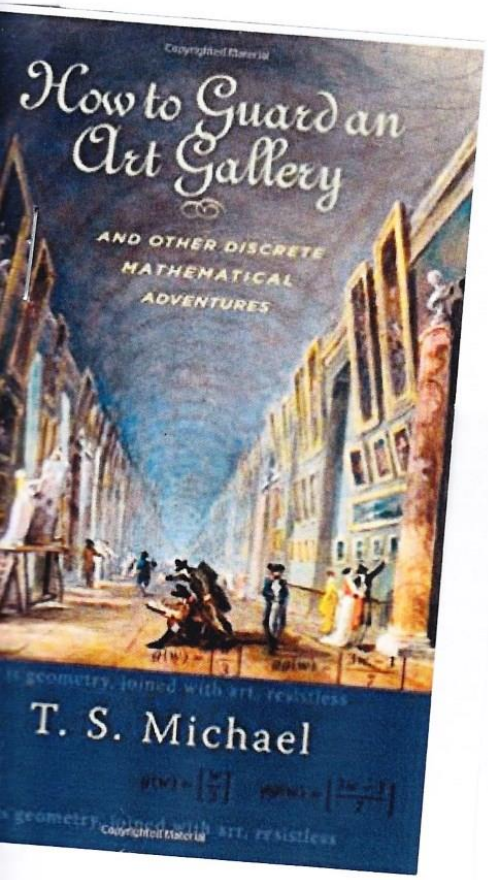
Fig. 4. $K_{4,4}$ minus two edges decomposed into two caterpillars.

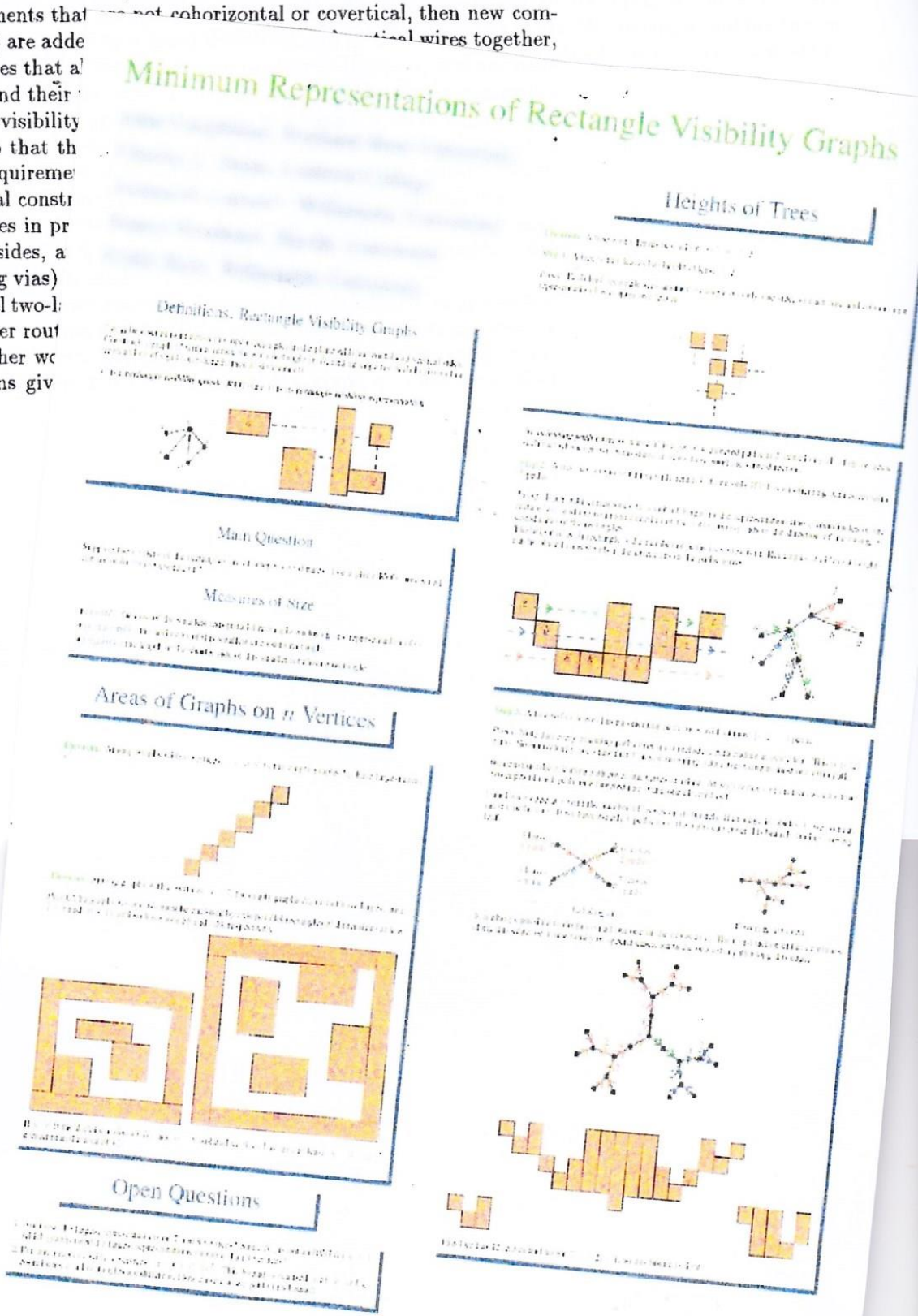
(with a slight extra condition) are noncollinear RVGs.
7. All graphs whose maximum vertex degree is three are noncollinear RVGs.
8. All graphs whose maximum vertex degree is four are weak RVGs.

A caterpillar is a tree containing a path with the property that every vertex is at distance at most one from the path. A high-degree vertex is a vertex of degree four or more.

The problem that we are studying has application to a type of VLSI design known as *two-layer routing*. In two-layer routing, one embeds processing components and their connections (sometimes called *wires*) in two layers of silicon (or other VLSI material). The components are embedded in both layers. The wires are also embedded in both layers, but one layer holds only horizontal connections, and the other holds only vertical ones. If a connection must be made between two components that are not cohorizontal or covertical, then new components (called *vias*) are adde ...tical wires together, resulting in bent wires that a... compared to wires and their ... graph is a rectangle-visibility ... can be embedded so that th... uses no vias. Our requireme... vated by the physical constr... similar problem arises in pr... naturally have two sides, a... equivalent of making vias)

The motivational two-l... ture of most two-layer rout... a specific size. In other wo... and their connections giv...





The paper consid...

they can see each other from up here and down here. They were an extension from bar visibility graphs, which have been studied for a lot longer. Then you can put restrictions on this, like studying unit bar visibility graphs where you can only use squares, and that restricts what you can do.

Y: OK. It looks like the rectangles in these diagrams weren't 'seeing' each other from above or below, though

C: Because we were doing trees, we don't have any cycles. So I think that's why we were trying to avoid using up-down visibility, you have to kind of limit it so you don't get a cycle. It's something you can use when you need three to see each other, like this.

X: Aah! Yes, I see. Right, so now we have this '97 paper. Here they say that their 'goal is to characterize those complete bipartite graphs that can be so represented…', So working out which of these vertices-and-lines kinds of graph can be represented as…

Y: Can be represented, like, we can build a rectangle graph that has the same number of vertices, related in ways that are parallel?

X: …right, can be represented as a rectangle visibility graph, uh, that's a – that's the kind of thing that people study.

C: Yes. And then we can say things about the properties of those rectangle visibility graphs, like how big they have to be to work.

Y: Also, I notice that these graphs have *lines*, rather than *arrows*.

C: Yes, that's right, so we're using an Oriented Path Cover like for example this poster that our research group made, that dealt with the area of the graphs.

Y: OK. Just out of interest, what do these… get used for? I mean applications?

C: A friend of mine who was also a student of my PhD adviser wrote a fantastic book called *How to Guard an Art Gallery* (Michael, 2009).[picks up book]. So you can imagine, a problem being that you have an art gallery with certain rooms, and you want the minimum number of guards, where do you position them so they can see all of the rooms…

X: Yeah, I see! That's great. Oh and this chapter here starts with a Georgia O'Keefe quote, 'I found I could say things with colours and shapes that I couldn't say any other way' – I guess that's about reasoning with diagrams?

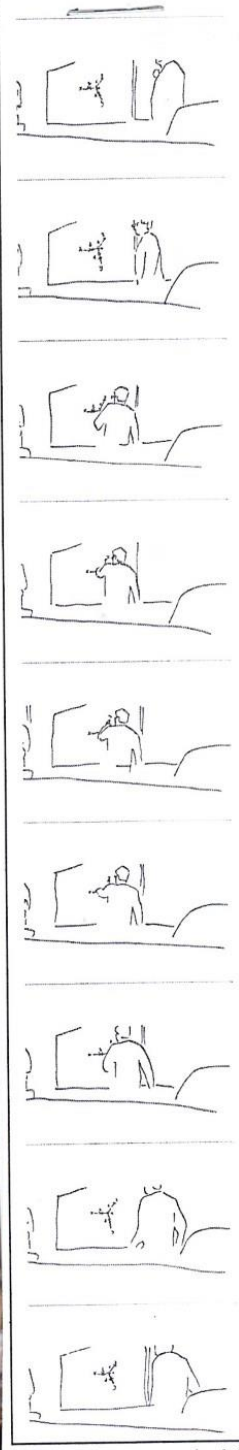
Y: Yes, though I note that diagrams are sometimes rightly treated with suspicion in mathematics. When you're looking at a particular diagram, the particular features in the example you choose might give you a misleading impression about the general claims that can be made.

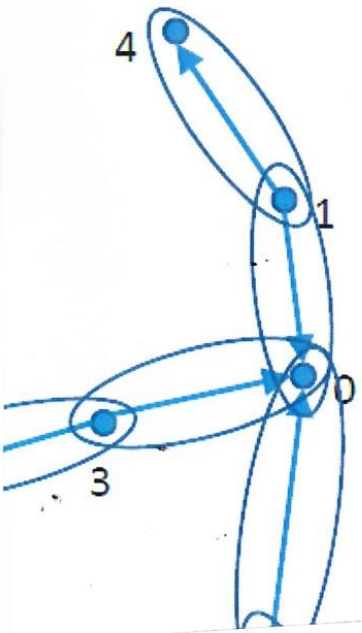
X: I notice in this '97 paper that reference is made to… VLSI design, that's kind of circuit board design, right? I guess that makes sense, as a useful application. Hey, thank you, C, that's been really helpful.

C: No problem!

Y: Yes, so let's reflect a moment—what kind of thing is it we've learned from talking to C?

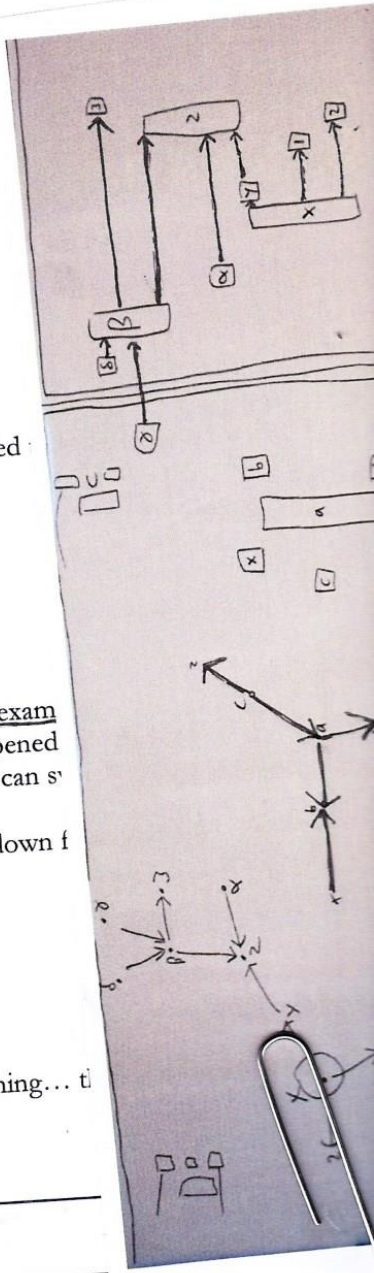
See GIF 1: *b* is switching for *a*…





An Orienting Path Cover for T:

1→0
2→0
3→0
1 → 4
2 → 5
3 → 6



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alright so the thing that's [bad
[OK I'm with you
… is a vertex with three switching vertices
[what
((nods)) [or more

or [more
[a - vertex with three switching vertices?
cos like [in that with that vertex *a*
[((unintelligible)) right there
… *b* is switching for *a*, [*d* is switching for *a* [and *c* is supposed
((walks over, follows path *xba*)) [yup ((follows path *zca*))[yup
but we have nowhere to put – *z*
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25 B: if there was… ((walks over to board, A steps away))
26 B: the problem is not for *x*
27 A: O↑H↓
28 D: ((unintelligible)) rectangle
29 B: if there exists a: swi- a ↑vertex↓ from which three vertices are switching… t
30 the whole graph's bad.
31 A: The whole graph's bad?
32 B: the- the tree cannot be represented if the vertex ((trails off))

X: Hmm. I guess we know a lot more about the history, who else has done what and why… the *why* seems quite important, I've been interested to learn what kind of questions people want to answer, what constitutes a contribution to this field.

Y: Yep! And questions we have, like why there are arrows instead of lines, and why in these examples we're just using left-to-right visibility, how it relates to the other work, those are hard to answer, because we don't have the right words to look for them. A broader picture of an ASSEMBLAGE might include the network of books, papers, online resources, and human repositories of knowledge.

3.1.5 Guiding Perception

X: OK, so let's get in to some of this detail. On lines 01 and 03, B proposes a STATEMENT of a definition of the restriction, a way to define which examples will be unrepresentable. C doesn't immediately accept the definition of the restriction

Y: Yep, and then F jumps in and talks through the example

X: So… A thing I want to talk about is this context that is shared by the group, this mutually manifest landscape. So that includes the – what's been said in the conversation so far, their memory of previous meetings… I know they had some items in their shared Dropbox that we've seen already, diagrams they'd made themselves and papers that they'd shared… then there's also what's on the board in the room…

Y: Yeah, so they have things they've all been looking at and thinking about. And a lot of it, they all know that they've all seen it, if you see what I mean, they *know* what they're all seen.

X: Right, but they still don't all *know* the same things. They have access to all of this shared material but they're only actually thinking about parts of it, and *which* parts might differ. There will be differences even if they had all seen the same material.

Y: Like if one person had really thought hard about one part, and then had been eating snacks when another was mentioned, or had worked on something else recently that was related…

X: Right, and so here (07) we see that B and F, and C, aren't quite on the same page.

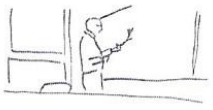
Y: OK, yeah. So then we can think a little about what F does to fix that. Let's look at this section… So F says that '*b* is switching for *a*, *d* is switching for *a* and *c* is supposed to be switching for *a*', and B comes over, and follows each line to show how it animates the diagram. So that's interesting, it's the *ordering* of F's explanation that clarifies that it is the *vertices*, not the paths, that are switching, because F says *b, c* and *d* first. Listing each of these letters locates the 'problem' in the points with these labels, as opposed to the paths. And it's – there's repetition, '*b* is switching for *a*, *d* is switching for *a* and *c* is supposed to be switching for *a*' with emphasis on b, d and c, so we see that it's the same each time, the emphasis is on the vertex. It's an ATTENTION-DIRECTING NARRATION.

X: Right! And then… so F simply lists each of the vertices, and B just gestures to the same paths. So… what they are doing is taking the static diagram, and animating it, making the first circle of vertices the focus and changing the role of the second circle to be 'things that need to be placed'.

Y: Yes, it's sort of… filling out the diagram, they are using their engagement with the diagram to shape C's understanding of it, and of the STATEMENT of a definition that B gave. So F doesn't try to explain B's condition. Instead F just tries to talk through the example to *exhibit* it as something that has certain features.

X: So… rather than further explicating the condition, F actually just identifies how example 1 meets it, the implication being that the example has three switching vertices and so cannot be represented. So F makes it clear that it's impossible in that case, and… stating it narratively like that shows that the

See GIF 2: So z was switching…

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ılright so the thing that's [bad
[OK I'm with you
.. is a vertex with three switching vertices
[what
((nods)) [or more
or [more
[a - vertex with three switching vertices?
cos like [in that with that vertex *a*
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.. *b* is switching for *a*, [*d* is switching for *a* [and *c* is supposed to be switching for *a*
walks over, follows path *xba*)) [yup ((follows path *zca*))[yup
ut we have nowhere to put – z ←.
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K what's your problem ((laughing))
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so if there was
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27 A: O↑H↓
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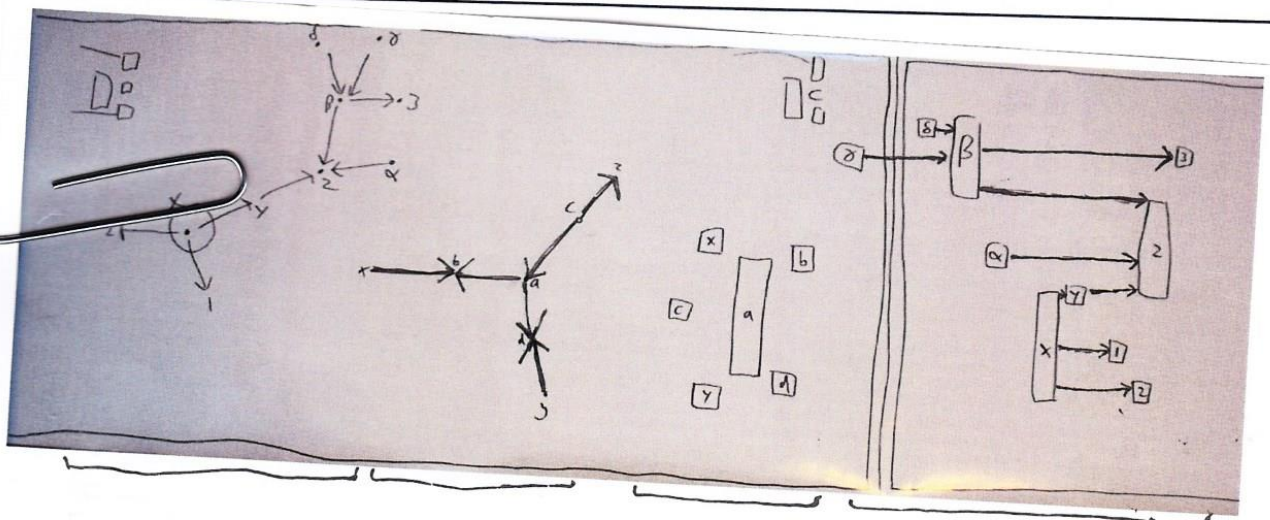

definition points to what it is that makes it impossible. You get to switching vertex 3, and it fails. And C can now see that.

Y: Yes, and the board notes are in this shared space, and they're almost *there for* people to interact with. We can tell who's holding the baton, so to speak, because they're standing next to the board.

X: Right, so you and I right now are making sense of these diagrams *by virtue of* the way they're used by the people, like that this one is exhibiting a property that needs defining, and this one *isn't* exhibiting that property, and they've been placed here so that people can interact with them, add bits and see what happens, find different ways to pay attention to different parts, physically interact with them—for their role in the ASSEMBLAGE. They're not just there to be decoded, they're external tools for reasoning.

Y: Yes, and it occurs to me that… Figuratively speaking, when STATEMENTS are proposed in lines 01 and 03, and 29, 30, 32, these are also placed in a shared space for the group to consider and critique, but are not set down on the board. The STATEMENTS can be discussed while they are fresh in the group's memory, and that might be why they're stated over and over again. Since the work of the group is to refine the STATEMENT, this is continuously helpful.

X: So this seems to involve a kind of sharing of mental states, a physical enacting of an understanding such that others can access the speaker's personal experience of what is going on and adopt it, or act upon it, change it. An *understanding* is a very difficult thing to directly share, and this acting-out-on-the-diagram allows something like *showing how you understand something* to occur through intentional highlighting and guidance. The speakers performatively think through the problem, and as they do it they talk through the process a little bit extra to make that thinking available to the watching participants.

3.1.6 Sharing Mental States

Y: OK, so then we have the 'problem is not for *x*', this part we're still not that sure about.

X: Yep. So in lines 20 and 22, we have this interesting action by A. [shows animation of lines 20-22]. A moves the focus back to the outer two diagrams by physically moving closer to this part. And A says, 'so… *z* was switching. … but what if it just continued this way and then switch' and follows the line up and then says 'but then a couple of things switched all at the' and seems to mime two coming down from above. So same here, we're seeing A sort of animate the diagram. A's talking about two coming down… I think the movements here are important, A follows the line up, so A's asking about the continuation of a switching path, what happens after it's switched once, because B has changed the focus to the *first* switch

Y: Hm, right. And then the 'two from above', A's actually talking about 'a couple of things switched all at the same time' so really I think that… A's asking a question about further switching downstream from an existing switching vertex. But A's kind of illustrating it by adding switching *paths*, not vertices. Because of this action, miming these two coming down from above, but that would only add one switching vertex.

X: Oh, I think I see what you mean. Yeah. But still, we can get the sense… I mean, A's worried about only counting this first one, about what happens afterward. It's still this question about *how to count* them.

Y: Ooh, yes. [slowly] *But* it's not – it's not as though A has *said* this, exactly, more like kind of … shared a way of looking at this diagram as having a path going up here and then things happening later on, and it's kind of vague, but still we can… kind of extrapolate from that what the miscommunication is

00.33.44.000

01 B: alright so the thing that's [bad
02 D: [OK I'm with you
03 B: … is a vertex with three switching vertices
04 C: [what
05 A: ((nods)) [or more
06 B: or [more
07 C: [a - vertex with three switching vertices?
08 F: cos like [in that with that vertex *a*
09 E: [((unintelligible)) right there
10 F: … *b* is switching for *a*, [*d* is switching for *a* [and *c* is supposed to be switching for *a*
11 B: ((walks over, follows path *xba*)) [yup ((follows path *zca*))[yup
12 F: but we have nowhere to put – *z*
13 B: right
14 A: I hav- I have a problem.
15 B: oerr gahd
16 B: OK what's your problem ((laughing))
17 E: ((laughing)) not again
18 B: ((speaking through laughter)) here comes another one of A's counter-examples
19 A: it just seems… ((points at diagram on board)) isn't… wait, what happened to this… so
20 A: so… *z* was switching. … [but what if it just continued this way and it can switch ((follows path))
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22 A: but then a couple of things switched all at the- ((mimes two coming down from above))
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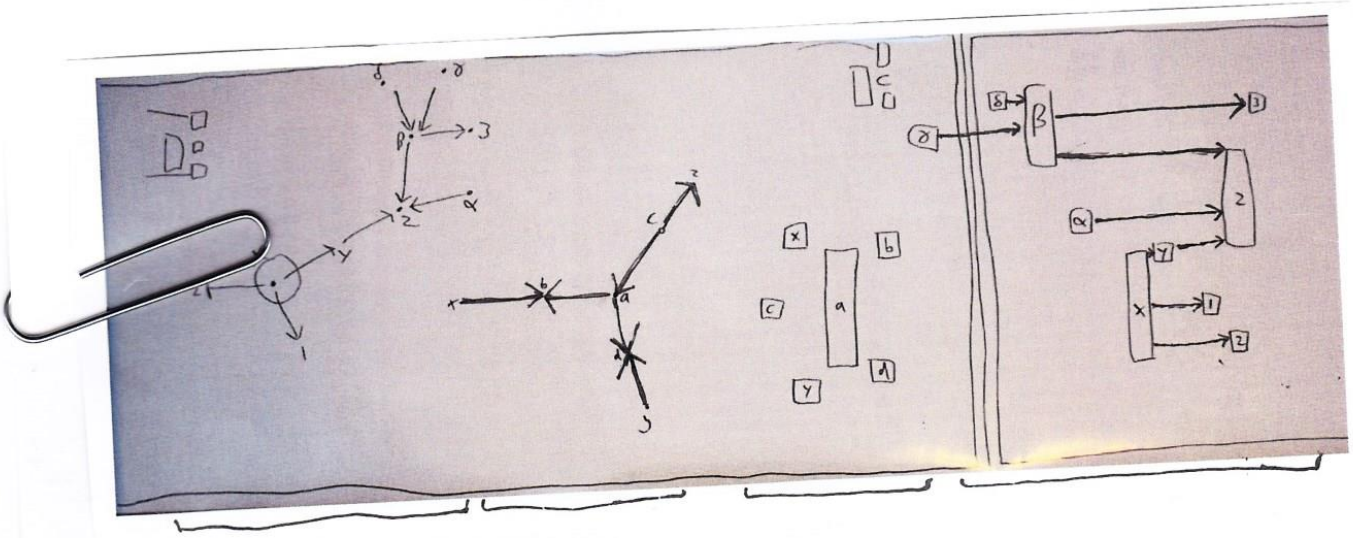

X: Right. And then B, B kind of thinks about it for a while and just says: [with emphasis] 'the problem is not *for x*'. This is great. Earlier on they had been using *x* as the vertex they were tracing paths from. But now… B's pointing out that it matters where you count it from, according to this new 'vertices' description, that if you introduced a bunch more switching vertices up there then if you were looking from *x* then it wouldn't look like a problem, but it *would* from another vertex. It's yet another interesting question about counting, about how to count

Y: Oh yeah! and I'm finding it funny that you're inhabiting these vertices now, you're giving them perspective

X: Yep, and A was kind of caught in *x*'s perspective, and B was trying to make the point that it doesn't matter which vertex it doesn't work for, as long as there's one it looks that way for

Y: OK yeah, so it's this kind of subtle question about how to 'count' them, which is a little hard to get at in words. It's really important for them to decide what gets counted, and what doesn't. You can have a diagram with a whole load of arrows switching directions, and you can still build the rectangle version, and you can have a diagram with just a few switches of direction and you can't build the rectangle. And they're just trying to find the right way of counting them, the right way to go through the diagram – really literally, it seems, we can see how it's helping to direct their attention around this drawn picture – and say, yes this one has too many of the *bad sort,* no, this one doesn't and should be fine

X: Yeah. And then A wasn't seeing that it could be counted in this other way. And so B just – just says something really brief to contradict the assumption that A was making.

Y: Right, so … rather than saying something like, 'you can start at any vertex', it's sort of as though B has just *perceived* A's picture of the problem as still being kind of… hooked on following the lines from *x*, and so B just contradicts that one assumption, and then everything sort of… hangs together in a different way for A

X: Right. This is a really tricky one, huh. I think we've more or less understood it, though.

Y: What I like is that neither A nor B really *say* what – neither of them really states anything clearly, the actual *words they utter* don't explicitly say any of this. They make points in these really indirect ways, and yet it clearly works perfectly.

X: Right, and… and if A hadn't gone through this process of thinking out loud with the diagram, or just thinking through it but doing so in a way that's audible and visible, then B could not have got such a sense of where A's understanding was at, and been able to know what to say to change it. THINKING OUT LOUD GIVES ACCESS TO MENTAL STATES.

Y: Yeah. So it really was as though A's thought process just happened, for that moment, externally, so that B could see it and replicate it. And so know how to alter it.

X: So after this exchange, after helping A to see the condition the way B is, then B restates it. B says 'if there exists a ↑vertex↓ from which three vertices are switching… then it's ↑bad↓. then the whole graph's bad.' So now emphasis is on that vertex *just existing*, wherever it might be. So now that B is confident that A has seen it, then B tries to state it as a sentence, to find the right STATEMENT.

3.1.7 Polysemy and Context

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01 B: alright so the thing that's [bad
02 D: [OK I'm with you
03 B: … is a vertex with three switching vertices
04 C: [what
05 A: ((nods)) [or more
06 B: or [more
07 C: [a - vertex with three switching vertices?
08 F: cos like [in that with that vertex *a*
09 E: [((unintelligible)) right there
10 F: … *b* is switching for *a*, [*d* is switching for *a* [and *c* is supposed to be switching for *a*
11 B: ((walks over, follows path *xba*)) [yup ((follows path *zca*))[yup
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16 B: OK what's your problem ((laughing))
17 E: ((laughing)) not again
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19 A: it just seems… ((points at diagram on board)) isn't… wait, what happened to this… so
20 A: so… *z* was switching. … [but what if it just continued this way and it can switch ((follows path))
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23 B: so if there was
24 A: °s that a problem?°
25 B: if there was… ((walks over to board, A steps away))
26 B: the problem is not for *x*
27 A: O↑H↓
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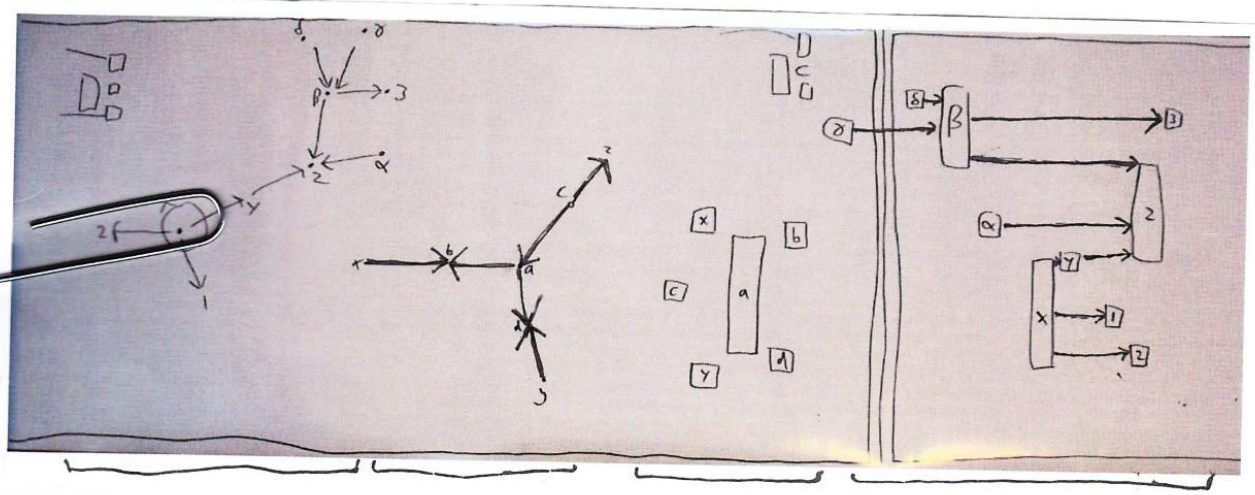


Y: Right. And there's this… 'it's bad, the whole graph is bad'. We should talk a bit about this term 'bad', this very emotive word that sort of refers to the work of the group but also has a particular sense, to do with difficulty with building one of these rectangle graphs that is parallel to a tree graph.

X: Yeah, so in lines 01-03 'bad' referred to the local problem happening in a particular vertex. The badness isn't so much something that is happening in the whole graph, but it *is* something that *can't be done* for the whole graph. The group want to be able to represent an entire graph, not just part—so if a part does not work, this counts as the entire example not working, for them.

Y: Yeah. So in these two lines (29 and 30), B aligns badness of one vertex with badness of the whole thing, but of a different kind. And A isn't – just asks for clarification, again by repeating word for word, like C did.

X: Right. B is already thinking in terms of what can or can't be represented, and evaluating everything relative to that goal; this is what B means by 'the whole graph's bad', which is understood at least by F. And that clarification, I think it… it answers the question by finishing the sentence in a more stateable way

Y: What do you mean by that?

X: I guess it's something that is clearer, more formal, less dependent on the group's informal shared understanding, more… portable?

Y: Hmm, interesting. So these clarifications- this time the STATEMENT is restated in a way that sounds like it's closer to the language of a paper. But the condition is only stated as being 'from which three vertices are switching', which is only one particular case, and they've already said that *anything* more than two will do it. So it's still stated in a way that's closely tied to a particular example.

X: OK. I just want to talk about valenced language for a moment. Like 'bad', and… They use the word 'problem' on four separate lines, on 14, 16, 24 and 26, but I get the impression that it means quite different things

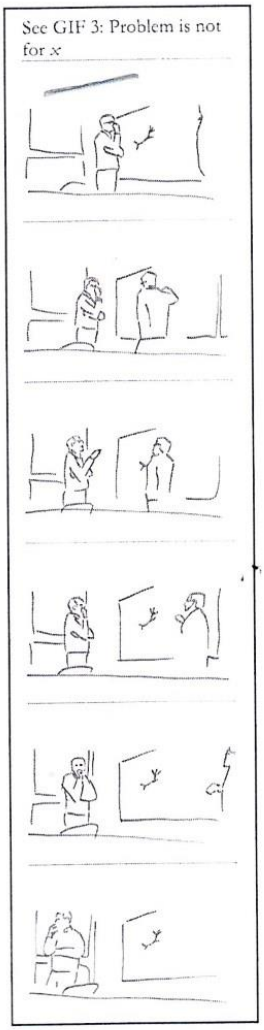
Y: OK, how so?

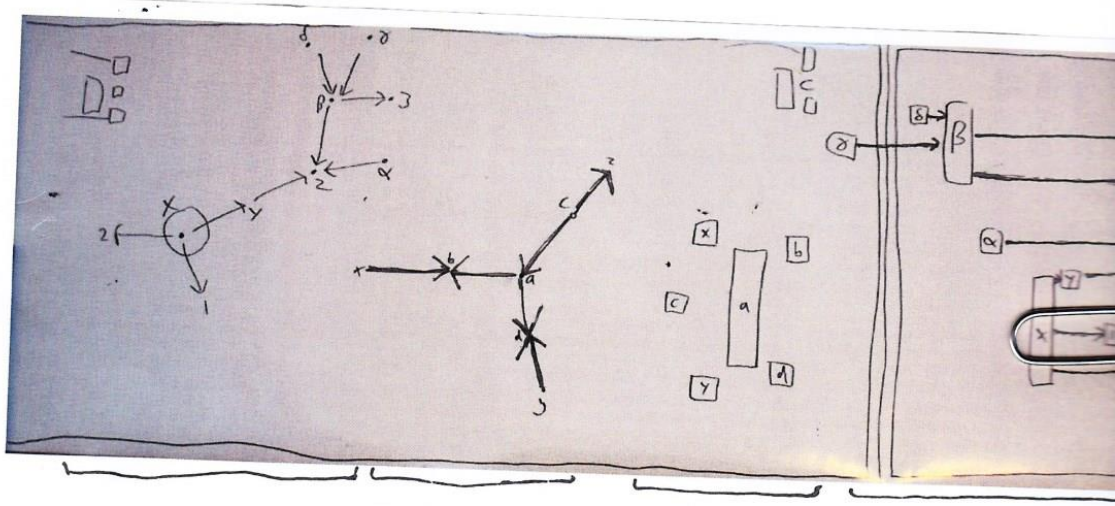
X: Well on line 14, A is expressing a concern about B's STATEMENT, and I think it's used in the same spirit on line 16 and 24, though 24 is less clear. But in 26, I *think* the 'problem' seems to be within the example, it's about what's unrepresentable so that the STATEMENT holds. And it seems on paper kind of incredible that these people successfully arrive at these different interpretations in such a short time.

Y: Sure. I mean, it's a common word, with lots of meanings. And when we hear it, we have to find the right one. I guess through context?

X: So that'll depend on things like how a word's been used recently, like a priming effect. But also in this context there are multiple different meanings in a short space of time, so there must be something else, something like what kind of answer you're expecting, what will answer the question, what makes sense in the context.

Y: Hm. OK, so let's look at the context of each a bit. So in each case, some kind of suggestion has just been tabled, so the- rather than 'hearer' let's say audience, since this stuff is kind of directed to the room, so the speaker knows that the audience expects effects related to that





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the whole graph's bad.
A: The whole graph's bad?
B: the- the tree cannot be represented if the vertex ((trails off))

context. In the first case, it's a problem with the STATEMENT that's just been presented to the room, and in the second it's a problem with representing the tree (which means that the STATEMENT holds).

X: Yeah. Their physical positions in the room seem to reinforce that. In 14, A is facing out into the room, and in 26 B has physically walked up to the diagram, B *physically moves* into the context of the diagram.

Y: So looking at the context the first two examples are talking about a flaw in *the definition of* unrepresentability, and the last one is talking about… a property that makes something unrepresentable. So if we try to pin down what's meant in the first, it's something like 'objection to the definition' or 'counter-example'—we can call that problem$_1$? And the final usage narrows down to 'unrepresentability', or 'specific locus of unrepresentability', and we can call it problem$_2$.

X: So they're really quite different, especially if you think about what's good for the group… the first usage, A's usage, really *is* a prob- going to cause difficulty for the work of the group, but B's usage is to show that because the diagram has a 'problem' then there *isn't* a new difficulty for the group's work! It seems like a heck of a perspective shift, to switch from what's a problem *for us* to what's a problem *for the diagram*

Y: Well, it's a bit more complicated than that because that second usage references the context of their previous work, wherein A proposed a counter-example that they described as a 'problem', that was a counter-example *because* it couldn't be represented. So the- there's a shared context in which the word 'problem' was recently used to refer to something that contradicted an expectation like problem$_1$, was unrepresentable like problem$_2$, *and* was a problem for the group's progress. And that old usage kind of encapsulates the two new narrowings, so they aren't *so* far apart.

X: Aah. So actually problem$_2$ is kind of an old problem, as far as the group's concerned. Ha. It's kind of… it's changed from a *problem* for them, frustration and all, into just a word to use to talk about a property of the diagram. It's like… they were in this world of trying to represent things as rectangle graphs, and in that world there was something that really was *bad* for their work, but now they've zoomed out into a world of trying to construct and evaluate STATEMENTS of definitions, and there's a new sense of 'problem' for the group, evaluated relative to the new aim, but the old usage still persists

Y: Like a sort of TELESCOPING, zooming out but LEAVING TRACES behind. Words like 'bad' or 'nice' pop up all the time in mathematics, with certain technical definitions. I wonder if this kind of thing is the reason.

X: Yes, and it strikes me that every time they are used, they still have this sense of referring to… people trying to do things, and that might even help people to connect with the content! To inhabit it, to get the sense of what people are trying to achieve, to get the sense of how something fits in with the aim of a piece of work. If a term hints at the role something played in that history of endeavour, then so much the better for the person reading and trying to make sense of it.

Y: Right. It seems relevant perhaps that there's evidence that really funny bits of polysemy can come into play in comprehension. There's been evidence that different but related meanings of words can 'prime' one another for faster comprehension (Williams, 1992), and that this effect is dependent on whether the meanings are related (MacGregor et al., 2015). And then there's research showing really surprising stuff that seems to relate interestingly to these ideas about

01.13.05.750

A: I'm starting to believe that this could be written [up as an algorithm

B: [that we have a proof of the lemma, that we have outlined the proof of the lemma

A: that- that- so if there are no 3-bad vertices is that what we're [calling them]

?: or four

?: [or four]

?: I dunno. Just if there [are no]

A: [just if there] are no k-bad vertices for >k greater than or equal to< THREE … then… th exists… >oh wait< then…

B: then this algorithm works to convert this orienting path cover into a rectangle

E: algorithm to be written

01 B: alright so the thing that's [bad
02 D: [OK I'm with you
03 B: … is a vertex with three switching vertices
04 C: [what
05 A: ((nods)) [or more
06 B: or [more
07 C: [a - vertex with three switching vertices?
08 F: cos like [in that with that vertex *a*
09 E: [((unintelligible)) right there
10 F: … *b* is switching for *a*, [*d* is switching for *a* [and *c* is supposed to be switching for *a*
11 B: ((walks over, follows path *xba*)) [yup ((follows path *zca*))[yup
12 F: but we have nowhere to put – *z*
13 B: right
14 A: I hav- I have a problem.
15 B: oerr gahd
16 B: OK what's your problem ((laughing))
17 E: ((laughing)) not again
18 B: ((speaking through laughter)) here comes another one of A's counter-examples
19 A: it just seems… ((points at diagram on board)) isn't… wait, what happened to this… so
20 A: so… *z* was switching. … [but what if it just continued this way and it can switch ((follows path))
21 B: [Yuh. I-
22 A: but then a couple of things switched all at the- ((mimes two coming down from above))
23 B: so if there was
24 A: °s that a problem?°
25 B: if there was… ((walks over to board, A steps away))
26 B: the problem is not for *x*
27 A: O↑H↓
28 D: ((unintelligible)) rectangle
29 B: if there exists a: swi- a ↑vertex↓ from which three vertices are switching… then it's ↑bad↓. then
30 the whole graph's bad.
31 A: The whole graph's bad?
32 B: the- the tree cannot be represented if the vertex ((trails off))

words with emotive content, like studies that show that positive-valenced words are understood more quickly when they're positioned high up, and negative when they're positioned low down (Meier & Robinson, 2004). That kind of thing can influence our interpretation in really subtle ways.

X: Yeah, and so… 'bad' started off being a value judgement, they were frustrated that the representation didn't work, and then right near the end of the meeting we see this kind of more formalised statement that they've settled on, which is still using the word 'bad'!

Y: Shall we take a look at that?

3.1.8 The Outcome

X: So they… as the meeting wears on, the group keeps stating and testing these sentences. And here they've come up with this, *the tree cannot be represented if there exists a vertex from which three vertices are switching.*

Y: OK, and do they refine it any more after that? It would be interesting to see what they are aiming for, if you see what I mean. The animal from the foliage.

X: At the close of the meeting, it's been refined into another line: I'm going to have to paraphrase a bit, but it goes like this: '*If there are no k-bad vertices for k greater than or equal to three then [a certain algorithm works to convert this orienting path cover into a rectangle]*

Y: And we think that k-bad is vertices that lead on to k switching vertices, where the switching vertex is the first vertex in the path to switch

X: Yeah, and so it's when they're more than 2-bad that's a problem

Y: What was that? 2-bad?

X: Oh, yeah, 2-bad [writes it down]! You could hear it as this [writes too bad] or this [to bad]… I think the group also had difficulty with that, spoken it could be any of those! I guess that just goes to show that really the... the aim is to produce something in text form, something written.

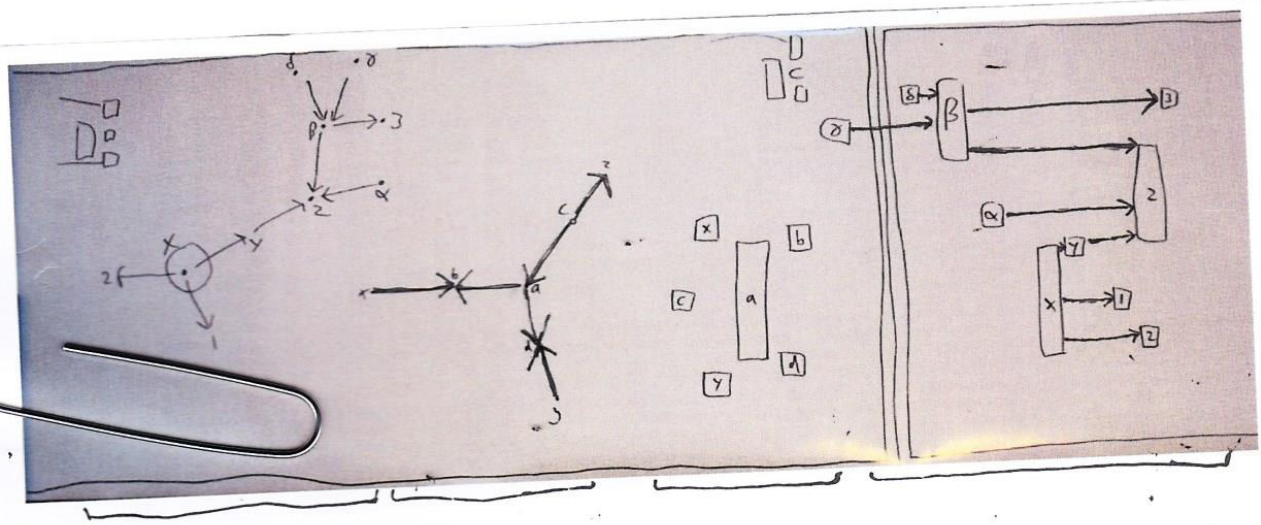
Y: Oh, that's interesting. WRITING TRUMPS SPEAKING.

X: Yes, and there's also… they talk all the time about producing a lemma, that the outcome is a lemma, right from here 12 minutes in (00.12.52.000) to right at the end (01.13.05.750). They're aiming for a written form, what they have in mind throughout is how to produce it a solidified, shareable form.

Y: Yeah. This is also the mirror-image of the draft STATEMENT passed around during the meeting, stated in terms of what needs not to be the case for the graph to be represented, rather than what needs to be the case for the graph to be unrepresentable.

X: Oh, that's interesting. And before they wanted to *exemplify* what can't be represented, so they could stare at it, and that would be three or more. And now it's a STATEMENT about what *can* be done as long as it *isn't* three or more.

Y: Why do you think that is?



00.33.44.000

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06 B: or [more
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13 B: right
→ 14 A: I hav- I have a problem.
15 B: oerr gahd
16 B: OK what's your problem ((laughing))
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20 A: so… *z* was switching. … [but what if it just continued this way and it can switch ((follows path))
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23 B: so if there was
? → *24* A: °s that a problem?°
25 B: if there was… ((walks over to board, A steps away))
26 B: the problem is not for *x*
27 A: O↑H↓
→ 28 D: ((unintelligible)) rectangle
29 B: if there exists a: swi- a ↑vertex↓ from which three vertices are switching… then it's ↑bad↓. then
30 the whole graph's bad.
? → 31 A: The whole graph's bad?
→ 32 B: the- the tree cannot be represented if the vertex ((trails off))

X: Well, because now the STATEMENT isn't *for* knowing what can't be represented, it's for knowing what *can*, because that's what they eventually want to do with the paper, that's what would interest, or could be used by, others, to show what *can* be represented.

Y: Oh, interesting. But now just this sentence is more kind of convoluted, with this negation at the beginning, the focus of the sentence being elsewhere. Lakatos talks about this in *Proofs and Refutations* (Lakatos, 1976), how definitions get adjusted and adjusted over time, they're the product of the efforts over time to find a proof. And this creates these intimidating, weird definitions where the thing you should focus on if you want to understand it is obscured. In this case, it's just that the STATEMENT no longer points *to* the examples they were discussing, it points *away from* them.

X: Yes, and I think that probably affects how easy it is to make sense of these STATEMENTS, and that can affect our reasoning. So for example: studies using the Wason selection task[2] have shown that, all things being equal, participants will tend to choose the cards that have been mentioned, because they're nudged in that direction by an assumption that their interlocutor will be mentioning the most relevant parts (Girotto et al., 2001; Sperber et al., 1995; Mercier & Sperber, 2011; Allott, 2002). The trouble there is that the formulations of the task aren't aligned with our usual means of communicating, and their artificiality pushes participants to make mistakes. So in the case of mathematics, these adjusted framings will include and exclude the cases that they want, but they might also become more and more effortful to read, since they may not immediately suggest the most central case, and instead introduce a lot of convoluted conditions that point in many other directions. When STATEMENTS are adjusted in a distributed way, the ATTENTION DIRECTION is sometimes left as a TRACE.

Y: That might cast some light on what it means for language to become more technical, rather than conversational. Specialised formulations that have been subjected to these purposive adjustments are less intuitively framed than everyday speech, or just diverge more from ordinary communication the more it happens, and that might mean that successful interpretation becomes more reliant on conventional familiarity.

X: Purposive like, with a *different* purpose than conversational communication. Like the usefulness to the community that we discussed. On the other hand I quite like the way that their emotive descriptions have been baked in, the wording that refers to the history of the group's work, using 'bad' as a term.

Y: Yeah. The description used is 'k-bad vertices for k greater than or equal to three', rather than simply 'greater than two'; this *does still* focus on the 'bad' cases in the same way that the meeting did, taking a particular case that was '3-bad' and examining it. It is interesting to look at the wording here, since there's nothing *necessary* about it- you could describe this problem in a multitude of different ways, like 'more than two folded vertices connected to one rectangle', 'maximum two double-ended vertices can be supported by one vertex'… If we used some inventive Greek-based terminology (from homo vs. hetero, and akri for tipped) we might say 'maximum two homoakrinous vertices can be held by one vertex'

X: At one point the group was calling them 'locations of incoherence'.

Y: Nice. But instead the language used throughout is that of switching, which- it's a term that worked really well with their earlier path-based description, it works really well for the perspective-taking method of counting that they talked about.

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[2] See footnote x in section x for a summary of the task, results and results from variations on the task

00.33.44.000

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16 B: OK what's your problem ((laughing))
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23 B:
24 A: °s that a problem?° so if there was
25 B: if there was… ((walks over to board, A steps away))
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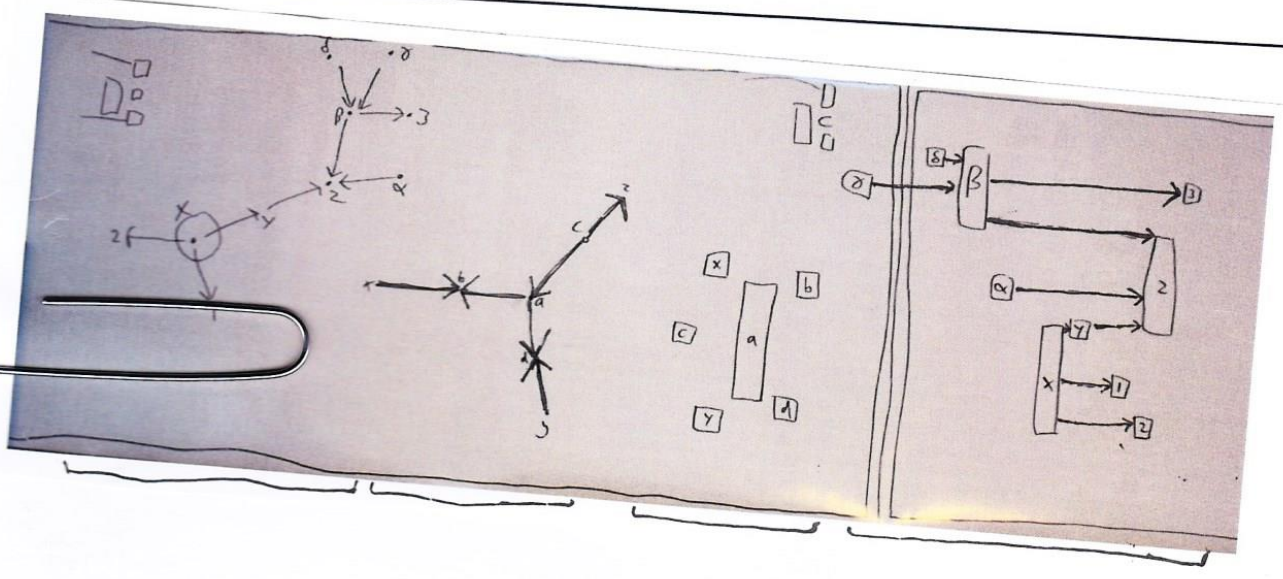

X: Yeah, so it's tied to the ways that these very human actors have been moving around these diagrams and how they've been helping one another to 'see' them, there's a TRACE LEFT BEHIND. The sentence has in it this kind of record of the group's discovery and management of a counter-example, even in quite subtle ways.

Y: That's an interesting way of seeing it. I suppose it's- if I were to describe it to you using the word 'switching', then maybe that already gives you a sense of following paths along and changing direction, more than 'folded' or 'double-ended' or 'homoakrinous'.

X: Yes, so retaining the word 'switching' might well be just enough to nudge a reader of the eventual paper into looking at a graph rather like the way they did in the meeting, and so reaching an understanding similar to theirs. And that isn't – it isn't that the language here *isn't technical enough* somehow, it's just in the nature of technical language, that it's borne from this stepwise refinement of language from situated, impressionistic ways of communicating gradually toward something more general and portable that can become part of the community's work. It could be expressed otherwise. But the very small details of its expression might help or hinder the creation of shared understanding in subtle and generally unseen ways.

Y: Hm. Especially if- so we basically think that the outcome of this work is this STATEMENT, right? That they were working toward this portable, repeatable STATEMENT, which was this *way to direct attention around* a whole set of possible examples, where they- they worked with these two examples, but they're supposed to *stand in* for all kinds of other graphs. And what they want is this short collection of words that'll help somebody to see things – 'the problem' – in the right way.

X: Yeah, though actually the reader of a paper might never even care that much about which examples are excluded. The STATEMENT might just draw a line around what can be done. But it *can be found* again.

Y: On the other hand, we were talking about how convoluted the STATEMENT had got, and Lakatos' observation about the development of definitions as they're passed around. And that's kind of unavoidable because mathematics is this collaborative activity across a whole *big* group.

X: Yes, the process of argument and counter-argument is greatly extended in the mathematical community. You see clarifications and counter-examples being traded over the course of many years, and definitions are greatly refined and developed by this process.

3.1.9 Summary

Y: OK, so are we finished?

X: I think so, more or less. We've gone through our own process of making sense of things, and tried also to make sense of how our subjects understood one another

Y: Which I guess is sort of a shortcut to seeing more clearly how we understood what they were doing.

X: Right. And how we read them, or read their actions in the shared environment, as part of the PEOPLE-ENVIRONMENT ASSEMBLAGE. And we found that the subjects DIRECTED ONE ANOTHER'S ATTENTION around the shared materials in really subtle ways, even when what they were trying to convey was not something they could have paraphrased, it was more like SHARING A MENTAL STATE, a way of seeing. And encapsulating the way of seeing as a STATEMENT was part of the work of bringing the animal from the trees.

00.33.44.000

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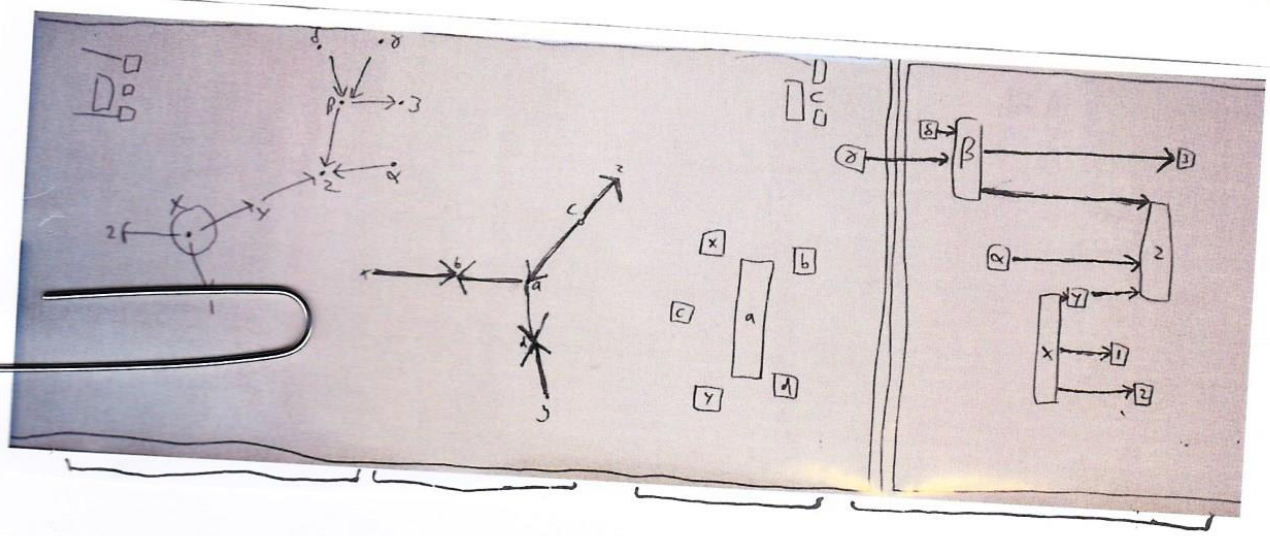

Y: Right, but then as the group zoomed out from one field of enquiry to a broader one, the STATEMENT kind of… solidified. The group still understood it, but this was rather more because they've repeated it and repeated it, they kind of *knew* it rather than *understood* it

X: Right, and the STATEMENT became more of an artefact than the genuinely communicative, contextual utterance that they began with. And this might explain the sense that we had that WRITING TRUMPS SPEAKING- if they were aiming toward a solidified, portable version in a paper then the expectation that the written form is the defining one makes sense. And yet, still it bore the TRACES of this genuinely mental-state-sharing function it once had.

Y: Right. And while we had access to the marks they'd made on the board, and the words they'd said, it wasn't just understanding the definitions of all of those that helped us to understand what was going on. We spent most of this time talking about where the attention of the group was, what was being emphasised, and working out what would make sense as an answer given the group's particular aims and motivations.

X: Yes! So we've seen that what is being communicated goes far beyond the content of the diagrams or even of the sentences uttered—they're using these really subtle strategies to shift each other's *way of seeing* the problem.

Y: Great! So shall we wrap up?

X: Sure! Til the next time.